11. Discuss the likely nature of the zero-power temperature coefficient and power coefficient for the following combinations of moderator and coolant for thermal reactors:

Moderator	Coolant
Graphite	Helium
Graphite	H_2O (liquid)
Graphite	H_2O (steam)
D_2O	H_2O (liquid)
D_2O	H_2O (steam)
D_2O	D_2O (liquid)

12. TREAT is a graphite-moderated, enriched-uranium pulsed reactor for testing fuel elements. The following specifications apply:

Temperature coefficient of reactivity = −2.4 cents/K
Specific heat (per unit volume) = 2.2 $MJ/m^3 \cdot K$
Prompt-neutron lifetime = 7×10^{-4} s

(*a*) Estimate the peak power density (W/m^3) resulting from a step reactivity input of 3.0 dollars. (*b*) Estimate the corresponding temperature increase assuming an adiabatic system.

Chapter 6

ENERGY REMOVAL

INTRODUCTION

THERMAL PROBLEMS IN REACTOR DESIGN

6.1. One of the unusual features of a nuclear reactor is that there is no theoretical upper limit to the rate of energy release, i.e., of power production, due to fission. In practice, however, the maximum power level of a reactor is normally determined by the rate at which the energy (heat) can be removed. Thus, in nuclear reactors operating at high neutron fluxes, such as those intended for central-station power or ship propulsion, the design of the core depends just as much on the heat-removal aspects as on nuclear considerations. The transfer of heat from fuel elements to the coolant is facilitated by increasing both the contact area and the coolant-channel volume. However, such increases generally result in a decrease of the multiplication factor of the system, so that additional fissile material is required to make the reactor critical. The heat-transfer and nuclear requirements may consequently be in conflict, and the actual design may represent a compromise between the opposing factors.

6.2. Most existing reactor-design concepts represent particular solutions of the heat-transfer problem in which the choice of coolant, the arrangement of fuel and coolant, and the method of heat removal are the primary considerations. Although each type of reactor has its own specific thermal problems, the solution of these problems can be approached in all cases by the application of conventional engineering principles of heat transfer, hydraulics (or fluid mechanics), and thermal stress. The equations needed to analyze the thermal behavior of a reactor are, therefore, those concerned with temperature distribution and thermal stress in solid components, and the flow of the reactor coolant. The term thermal-hydraulic design is commonly used to describe the effort involving the integration of heat transfer and fluid mechanics principles to accomplish the desired rate of heat removal from the reactor fuel. The purpose of this chapter is to examine the basic aspects of thermal-hydraulic design.

6.3. Before considering the individual problems of heat removal, however, mention should be made of some unusual thermal features of nuclear reactors. Since heat is being continuously produced by fission and other nuclear processes, a path must be provided for its transmission to a sink to prevent a steady temperature increase. Unlike a conventional power plant, where the temperature is limited to that resulting from the combustion of coal, oil, or gas, the temperature in a nuclear reactor could increase continuously until the reactor is destroyed if the rate of heat removal were less than the rate of heat generation. An example of this type of behavior would be an uncooled fuel element in which the heat from a sustained fission reaction, or from radioactive decay, raises the temperature to the point where damage occurs. The rates of heat generation and of heat removal must therefore be properly balanced in an operating reactor.

6.4. For a reactor of given design, the maximum operating power is limited by some temperature in the system. This limit may be set by changes in the properties of the fuel elements, the coolant, or the moderator, by allowable thermal stresses in some parts of the system, by the influence of temperature on corrosion, or by other thermal effects. The maximum permissible temperature must be definitely established to make sure that the cooling system is adequate under all anticipated operating conditions. This is done by, first, estimating the magnitude and distribution of the heat sources in the reactor and, second, determining the temperature differences along the various paths of heat flow in the system. These two interrelated areas of investigation form the basis for both the concept of a new reactor system and the thermal analysis of a specific design.

6.5. An important aspect of thermal-hydraulic design is concerned with conditions that might arise from an accident. Provisions must be made in the design to accommodate deviations from normal operating conditions, such as would occur following partial or complete loss in the coolant flow. Further analysis is required for the design of safety features which are provided in a reactor plant to reduce the consequences of a serious accident in which the core might be damaged. In order to discuss this subject adequately, it is necessary to understand the phenomena associated with a loss-of-coolant accident and the design of the safety system. Consequently, the treatment will be deferred until Chapter 11.

GENERATION AND DISPOSAL OF HEAT IN REACTOR SYSTEMS

6.6. The determination of the manner in which the heat sources are distributed in the reactor core and structure is complicated by the fact that the heat is generated in a number of different ways. The major portion arises from the kinetic energy of the fission fragments which usually manifests itself as heat released entirely within the fuel elements, but heat is also produced from the slowing down of neutrons and beta particles, and the absorption of various gamma radiations. Since the neutrons and gamma rays are not uniformly distributed throughout the reactor core and structure, the associated heat-source distribution will also be nonuniform. An accurate knowledge of neutron and gamma-ray fluxes is therefore necessary for a complete solution of the reactor cooling problem. These fluxes are difficult to estimate with sufficient accuracy for thermal calculations, and so, in the final analysis, the reactor designer must frequently rely upon data obtained from experiments or from operating reactors.

6.7. Once the heat-source distribution is known, the determination of the temperature distribution in the reactor and of the temperatures of the various coolant circuits leading to the heat sink can be made in a straightforward manner. Here the main objective is to follow the heat flow from the fuel element to the coolant and from the coolant to a sink. Although a heat sink must be provided in all cases, the heat-flow path will vary with the nature of the reactor system. In some cases the heat is discharged directly from the primary coolant to a sink, e.g., a river or the atmosphere. In other instances, a secondary or even a tertiary coolant may be used to transfer heat from the primary coolant to the sink. Further reference to this aspect of reactor design will be made shortly.

6.8. The temperature at any point in a reactor will be greater than that of the sink by an amount equal to the sum of all the temperature drops along the heat-flow path. The temperature drops will usually be proportional to the amount of heat flowing and, hence, to the reactor power. Thus, the steady-state temperature in any component is generally determined by the sink temperature, the reactor power level, the total power generated in the component, and the effective thermal resistance between the component and the sink.

SPECIAL THERMAL PROBLEMS

6.9. In addition to the manner in which heat is generated and disposed of in a reactor, there are other special problems unique to reactor design that must be considered. There is, in particular, the matter of selecting reactor materials and coolants. In a conventional heat engine or power plant, materials of construction are chosen on the basis of mechanical performance, but in a reactor system the choice is dictated, to a considerable extent, by nuclear properties. This often results in the selection of unconventional, high-cost materials with their attendant problems. Such materials do not always have the most desirable thermal, physical, or mechanical properties, and frequently these properties are not well known. Beryllium metal, for example, is an excellent material for use as moderator and reflector, but it is relatively brittle (§7.142). Austenitic stainless steels are used as the cladding for fast reactor fuels, but they tend to swell as a result of exposure at high temperatures to fast neutrons (§7.104).

6.10. Another factor which adds to the problem of heat removal from the reactor core is that the volumetric heat-release rates (or power densities) tend to be quite high. That is to say, large amounts of heat are released (and must be removed) per unit volume. In pressurized-water and boiling-water reactors, the amount of water moderator, and hence the core size, are limited in order to minimize neutron capture; and fast reactors are relatively compact in order to reduce both the cost of the fuel

charge and neutron moderation. In any event, an economical design is achieved by extracting as much energy as is practical from a reactor core of given size. Furthermore, the cost of reactor components, such as the pressure vessel and shield, depend on the size. Because of these considerations, the power densities in pressurized-water reactors are approximately 100 MW (thermal)/m^3 and about 55 MW (thermal)/m^3 for boiling-water reactors. A typical sodium-cooled commercial fast breeder reactor is expected to have a power density as high as 500 MW (thermal)/m^3. By contrast, the heat released in a forced convection, fossil-fuel fired steam boiler is of the order of 10 MW (thermal)/m^3.

6.11. The combination of high operating temperatures (for maximum thermal output of the reactor and maximum thermodynamic efficiency for ultimate conversion of heat into power) and large power densities makes special demands upon the design of the reactor heat-removal system. Not only do the high power densities cause stresses due to temperature differences in thick core components, but the nuclear radiations at these power densities, i.e., high neutron fluxes, may have adverse effects on the thermal conductivity and other properties of reactor materials (Chapter 7). In general, the major advances in reactor design have come from material and fabrication developments, but it is only by optimizing the reactor systems from the thermal-hydraulic standpoint that their full advantage can be realized.

THERMAL TRANSPORT

6.12. The goal of the reactor thermal-hydraulic design is to provide for the "optimum" transport of heat from the point of deposition in the fuel to its conversion into useful energy, normally in a turbine. By "optimum" is meant a proper balance between many opposing design parameters, such as coolant flow rate, pressure loss, temperature distribution in the core, material properties, etc. The objective is to remove the heat economically from the fuel without exceeding various constraints that might lead to a failure of the materials.

6.13. Several different fluid loops may be involved in the transport of heat from the reactor fuel to the turbine (§1.92 et seq.). For example, in a pressurized-water system, a steam generator serves to transfer heat from the primary coolant to the steam which is expanded in the turbine. In liquid-metal cooled, fast breeder reactors, heat is transported from the primary sodium coolant to secondary sodium, and then in turn to steam in a steam generator. In practically all reactor systems, an additional thermal transport step is required after the steam is expanded in the turbine since the "waste" energy must be dissipated to a sink, normally by transfer to condenser water.

HEAT-SOURCE DISTRIBUTION

6.14. In the design of the coolant system, it is important to know the distribution of heat sources and in turn the temperature pattern in the core. Some discussion of

these considerations is given in the following sections preliminary to a review of the heat-transmission principles upon which heat removal depends.

6.15. In a reactor using solid fuel, the design of the fuel elements and of the spacing of coolant passages through the various solid components is influenced by the temperature distribution within these components. The first step in the determination of this distribution is to establish the maximum volumetric heat-release rate in each of the components being considered. The procedure for doing this will be discussed later. Although the heat-source distribution depends upon the particular reactor design, there are certain generalizations which can be used for preliminary calculations, at least.

6.16. It may be assumed that, in a thermal reactor, 90 percent of the total heat is liberated in the fuel elements and that this heat source will be distributed in a manner proportional to the fission rate. About half of the remaining heat, i.e., 5 percent of the total, will be released in the moderator, and the remaining 5 percent in the reflector and shields. The moderator heat-source distribution will be somewhere between a uniform distribution and one proportional to the thermal-neutron flux. In many reactor designs, each coolant channel takes up heat from both the fuel element and the surrounding moderator, and in these cases the two heat sources are combined.

6.17. The determination of the heat-source distribution in the reflector and shields is somewhat complicated, because it involves an estimate of the neutron leakage from the core and of the gamma-ray flux as a function of the energy spectrum. Since gamma rays arise from a number of sources, e.g., fission, fission-product decay, and radiative capture (cf. §10.13 et seq.), the details of each originating nuclear process must be known, and the subsequent behavior of the radiations calculated. Although only 5 percent of the total heat is released in the reflector and shields (including the pressure vessel if one is used), the cooling of these components may be a significant problem in a reactor operating at high power.

THERMAL DESIGN PROBLEMS

6.18. A common second step in analyzing or designing the cooling system of a reactor is to calculate the temperature distribution in the solid components from their known heat-source distributions, thermal conductivities, and configurations, using appropriate heat-flow equations. Since fuel elements are usually in the form of long cylinders (or rods), a preliminary calculation of the heat flow and temperature distribution can be made in a relatively simple manner, as will be seen subsequently.

6.19. Solid moderator and reflector components, on the other hand, do not always have simple configurations; moreover, the spacing of cooling channels in the shields may not be uniform, because the heat liberated falls off with increasing distance from the core. Such cases can often be treated by simplifying the geometry to a form, e.g., a cylinder, which permits the use of an established heat-flow equation. Alternatively, a point-to-point analysis can be made using computer techniques (§6.77).

6.20. Fixing the temperature distribution pattern in the core and the design of the cooling system require evaluation of the flow characteristics and heat-transfer coefficients of the coolant. These determine the temperature drop between the surface of the solid fuel which is being cooled and that of the coolant, and also of the temperature in the coolant stream. As will be seen shortly (§6.46 et seq.), the difference in temperature between the fuel surface and the adjacent coolant determines the rate of heat transfer (per unit area) from the fuel to the coolant. The flowing coolant then transports the heat to another location. The amount of heat transported (per unit mass of coolant) is related to the increase in temperature of the coolant and its specific heat; the heat-transport rate is obtained upon multiplying the heat transported per unit mass of coolant by its mass-flow rate (mass per unit time). The temperature increase of the coolant flowing through the hottest channel is of special significance, since it establishes the maximum coolant temperature in the reactor.

6.21. Finally, it is necessary to consider the transfer of heat, in stages, from the primary coolant to the sink. Such transfer may involve heat exchange from the primary coolant to a steam system, or from the primary coolant to a secondary coolant and then to a steam system, and ultimately to a heat sink. This aspect of the subject is treated along more or less conventional lines.

6.22. In addition to the matters already discussed, there are a number of auxiliary problems, related to the thermal characteristics of a reactor, that have a bearing on the design of the cooling system. One of the most important of these is the problem of stress in solid components due to temperature differences. For example, problems of stress may arise in the design of thermal and biological shields because of the considerable differences between average and surface temperatures. Furthermore, in pressurized-water reactors, thermal stresses in the thick-walled pressure vessel become very significant. This subject will be treated in Chapter 7.

6.23. The coolant itself often presents special problems that will be considered in a later section of this chapter. However, mention may be made here of such considerations as purification, induced radioactivity resulting from neutron capture, decomposition by radiation, and corrosion (or erosion) including mass transfer. These are important aspects of the design of the thermal system of a reactor, but they primarily represent problems in materials and equipment rather than of heat transfer.

6.24. In summarizing the foregoing discussion it can be seen that, apart from problems associated with thermal stresses and the choice of coolant, the design (or analysis) of the cooling system of a reactor involves the following topics:

(1) Heat sources and their distribution.
(2) Heat conduction through solid components and to the primary coolant.
(3) Heat transport along reactor channels due to flow of fluid.
(4) Heat exchange from primary to secondary coolant and to the steam-generating system (or heat sink).
(5) Heat transport during transient (accident-related) conditions.

HEAT SOURCES IN REACTOR SYSTEMS

FISSION ENERGY

6.25. The energy released in the core by fission appears in various forms, but mainly as the kinetic energy of the fission fragments, the fission neutrons, and the beta particles resulting from radioactive decay of the fission products. The fission fragments are usually stopped within the fuel elements themselves; the small fraction that escapes into the cladding penetrates only about 0.01 mm. The beta particles of high energy may travel up to 2 mm in a cladding material such as zircaloy (§7.114), and so a large fraction of these particles may escape from the fuel element into the moderator or coolant, but they will not get out of the reactor core. The fission neutrons lose most of their energy in the first few collisions with moderator atoms, and they travel distances ranging from some centimeters to a few feet. It is seen, therefore, that most of the heat from the three sources under consideration, comprising about 90 percent or more of the total energy generated, will be released within the reactor core.*

6.26. The remaining 10 percent (or less) of the energy produced in fission appears as gamma rays from the several sources mentioned earlier. The manner in which these are distributed throughout the reactor core, reflector, and shields depends upon the materials present and the configurations of the various components; the distribution will consequently depend upon the reactor type. A complete analysis of the heat generation due to gamma radiation is complicated; it requires a knowledge of the energy spectrum and spatial distribution of each of the gamma-ray sources, as well as the absorption coefficients of the materials in the system for radiations of different energies.

6.27. In addition to the spatial distribution of heat sources, the time dependence must also be considered since an appreciable proportion of the energy of fission is released over a period of time. For example, the fission fragments may lose essentially all of their kinetic energy in a few microseconds, whereas the energy accompanying the radioactive decay of the fission products is released over many years. Actually, the approach to equilibrium thermal conditions after the startup of a reactor is very rapid, and any departure from this equilibrium can usually be neglected.

6.28. Provision must be made, however, for cooling the fuel elements and other reactor components for some time after shutdown because of the heat generated in the decay of the fission products (cf. §2.213). The heat source distribution will then be different from that when the reactor is in operation. One hour after shutdown, for example, the heat generation rate in the fuel elements will be about 1.5 percent of the operating value, whereas in the reflector and shield it will be approximately 10

*According to the statement in §6.16, the heat released in the core is about 95 percent (90 percent in the fuel elements and 5 percent in the moderator) of the total. The additional 5 percent or so arises from gamma-ray absorption elsewhere.

percent of the rate during operation. This difference arises because a large fraction of the heat released after shutdown is due to absorption of gamma rays from fission-product decay. Although this is a small proportion of the heat generated in the core during operation, it makes a considerable contribution to the heat liberated in other parts of the reactor system.

6.29. It was seen in Chapter 1 that the total energy released in fission, which ultimately appears as heat, is made up of contributions from a number of sources. In general, the total energy, exclusive of the neutrino energy which is lost to the reactor system, may be expressed by

$$E \approx 191 + E_c \text{ (in MeV)},$$

where E_c is the energy liberated as a result of various parasitic neutron capture processes, e.g., nonfission capture in uranium-235 and uranium-238, and capture in moderator, coolant, structure, etc.; this includes the energy of the capture gamma radiations and the decay energies, i.e., the energies of the alpha and beta particles and gamma rays, of any radioactive species that are formed by parasitic neutron capture.* Since the value of E_c will obviously depend upon the nature of the materials present in the reactor core, it is evident that the total amount of heat produced by fission will vary, to some extent, from one type of reactor to another.

Example 6.1. Determine the total energy release in the core of a pressurized-water reactor (PWR) having volume fractions of uranium oxide (UO_2), water, and iron of 0.32, 0.58, and 0.10, respectively. The oxide fuel (density = 10.2×10^3 kg/m^3) has an average enrichment of 2.8 percent ^{235}U, and the average cooling water density is 0.69×10^3 kg/m^3. The energy released per neutron captured in uranium, water, and iron may be taken as 6.8, 2.2, and 6.0 MeV, respectively.

The solution requires the calculation of the macroscopic neutron capture cross section for each component (except for the oxygen in UO_2 which is very small) as well as the ^{235}U macroscopic fission cross section. The final result depends on ratios of cross sections rather than absolute values; hence, the tabulated values for 0.0253-eV neutron microscopic cross sections may be used in the calculations. The capture-to-fission ratios are then as given in the following summary:

	^{238}U	^{235}U	H_2O	Fe
N (nuclei/m$^3 \times 10^{28}$)	0.71	0.020	1.34	0.85
σ_c(b)	2.7	98.6	0.664	2.55
σ_f(b)	–	582	–	–
Σ_c(m^{-1})	1.9	1.97	0.89	2.2
Σ_f(m^{-1})	–	11.6	–	–
$\Sigma_c/^{235}\Sigma_f$	0.16	0.17	0.076	0.19

*Although E_c should include the decay energies of all radioactive species formed as a result of nonfission neutron reactions, only those of relatively short half-life contribute to the energy release in the state of pseudo-equilibrium attained by the reactor soon after startup.

The energy released as a result of nonfission captures is

$$E_c = (6.8 \times 0.16) + (6.8 \times 0.17) + (2.2 \times 0.076) + (6.0 \times 0.19)$$
$$= 3.6 \text{ MeV}$$

The total energy released per fission in this core is thus

$$E = 191 + 3.6 \approx 195 \text{ MeV} = 3.1 \times 10^{-11} \text{ J}.$$

SPATIAL DISTRIBUTION OF ENERGY SOURCES IN REACTOR CORE

6.30. As already noted, a large proportion of the energy from fission is available in the form of heat within a very short distance of the fission event. The total rate of heat generation is proportional to the fission rate, i.e., to $\Sigma_f\phi$ or $N\sigma_f\phi$, where σ_f is the microscopic fission cross section, ϕ is the neutron flux, and N is the number of fissile nuclei per unit volume of fuel. If N remains uniform throughout a particular reactor or reactor region, as an initial approximation, the thermal source function, expressed as the power density, may be taken to be the same as the spatial distribution of the neutron flux. Local perturbations of the flux, such as occur within a fuel rod in a heterogeneous reactor, normally have little effect on the overall temperature distribution and so they can be ignored.

6.31. The overall flux distribution in the reactor core can be calculated by the methods described in Chapters 3 and 4. For a reflected cylindrical reactor in which the fuel is distributed uniformly, the flux distribution in the core and reflector can be represented approximately by

$$\frac{\phi}{\phi_{\max}} = J_0\left(2.405\frac{r}{R'}\right)\cos\frac{\pi z}{H'}, \tag{6.1}$$

where $\phi_{\max}$ is the flux at the center of the core where it is assumed to have its maximum value; J_0 is the zero-order Bessel function of the first kind, r and z are the radial and vertical coordinates, as in Fig. 3.11, and R' and H' are the effective radius and height, respectively, of the reactor, including an allowance for the reflector.

6.32. The average neutron flux in the core is given by

$$\phi_{\text{av}} = \frac{1}{\pi R^2 H}\int_0^R \int_{-1/2H}^{1/2H} \phi 2\pi r\, dr\, dz, \tag{6.2}$$

where R and H are the actual radius and height of the fuel-containing region. Upon substituting the expression for ϕ from equation (6.1) and performing the integration, the result is

$$\frac{\phi_{\text{av}}}{\phi_{\max}} = \frac{2RR'J_1\left(2.405\frac{R}{R'}\right)}{2.405R^2} \cdot \frac{2H'\sin\left(\frac{\pi}{2}\cdot\frac{H}{H'}\right)}{\pi H}, \tag{6.3}$$

where J_1 is the first-order Bessel function.

Gap Conductance

6.72. As shown in Fig. 6.6, fuel rod designs normally provide for a small annular gap between the uranium dioxide fuel pellet and the cladding. The gap is initially filled with helium, but during irradiation there is some dilution of the helium with fission-product gases. More important, however, is the swelling and cracking of the pellet (§8.85) which tends to close the gap in a nonuniform manner. The interface heat-transfer problem therefore progresses from one involving conduction through the gas to a combination of conduction through the gas and across the interface between two surfaces in partial contact. Although analytical models have been developed based on the thermal conductance of a gas film of the thickness indicated by the roughness of the two surfaces, the results tend to be unreliable. Semiempirical computer models based on test data are therefore used for design purposes [1]. The gap conductance is also important in the calculation models required for safety evaluation (§11.97). For orientation purposes, the order of magnitude of the heat-transfer coefficient at the interface of irradiated oxide pellets and metallic cladding may be taken to be 6000 $W/m^2 \cdot K$.

Example 6.3. A PWR fuel rod (see Example 6.2), with pellets 8.19 mm in diameter, is clad with zircaloy 0.57 mm thick; the outer diameter of the clad rod is 9.5 mm. If the bulk (mixed-mean) coolant temperature is 315°C and the volumetric heat generation rate (power density) in the fuel is 3.20×10^8 W/m^3, determine the temperature at the center of the fuel and at the outer surface of the cladding. The heat-transfer coefficient at the cladding-coolant interface may be taken to be 34 $kW/m^2 \cdot K$. The thermal conductivity of the zircaloy is 13 $W/m \cdot K$.

The temperature drop between the coolant and the cladding is obtained from the second term on the right of equation (6.22); thus,

$$t_2 - t_m = \frac{Qa^2}{2hb},$$

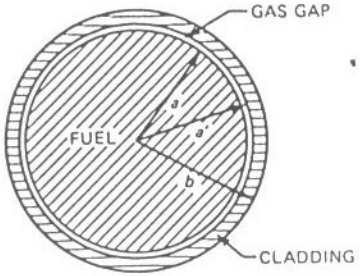


Fig. 6.6. Section through fuel pellet with gas gap and cladding.

where

$$Q = 3.20 \times 10^8 \text{ W/m}^3$$

$$a = \tfrac{1}{2}(8.19 \text{ mm}) = 4.095 \text{ mm}$$

$$b = \tfrac{1}{2}(9.50 \text{ mm}) = 4.75 \text{ mm}$$

Hence,

$$t_2 - t_m = \frac{(3.20 \times 10^8)(0.004905)^2}{(2)(3.4 \times 10^4)(0.00475)} = 24°\text{C}.$$

Since t_m is 315°C, the cladding surface temperature is $315 + 24 \approx 340$°C.

The temperature drop across the cladding is obtained from the first term on th right of equation (6.22); that is,

$$t_1 - t_2 = \frac{Qa^2}{2k_c} \ln \frac{b}{a'},$$

where, as shown in Fig. 6.6, a' is the inner radius of the cladding, i.e., $\frac{1}{2}(9.50)$ $0.57 = 4.18$ mm. Hence,

$$t_1 - t_2 = \frac{(3.20 \times 10^8)(0.004905)^2}{(2)(13)} \ln \frac{0.00475}{0.00418} = 38°\text{C}.$$

(Use of the approximation for $\ln(b/a')$ in §6.58 footnote leads to essentially th same result.)

Next, consider the temperature drop Δt across the gas gap between the fuel ar the cladding. This is given by an expression equivalent to the second term on th right of equation (6.22), except that b is replaced by a and h by the gap conductanc

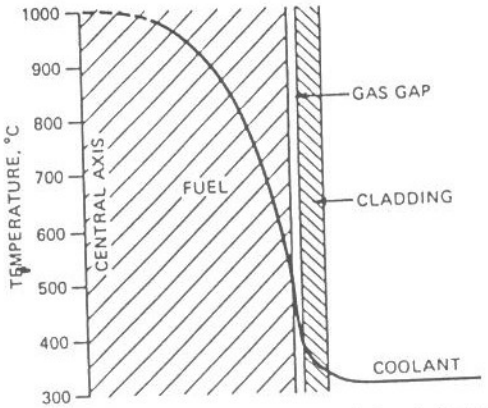


Fig. 6.7. Approximate radial temperature distribution in a fuel rod of a water-cooled reacto

pending on flow conditions and other parameters, and so it is an important consideration both in the initial reactor design and in the analysis of the consequences of various accident possibilities. For example, an increase in resistance, corresponding to a decrease in the heat-transfer coefficient, would result in an increase in the temperature difference between the cladding and the bulk of the coolant if the heat flux were to remain constant. If the coolant temperature is not to change, the net result would be an increase in the temperature at the outer surface of the cladding, and ultimately in the temperature of the fuel, as can be seen from equation (6.21) et seq. Thus, an examination of the considerations that affect the convection heat transfer is important in reactor design.

LAMINAR AND TURBULENT FLOW

6.92. When a fluid flows through a straight pipe at low velocity, the particles of fluid move in straight lines parallel to the axis of the pipe, without any appreciable radial motion. This is described as laminar, streamline, or viscous flow, and the "velocity profile" is depicted diagrammatically in Fig. 6.11A; the lengths of the arrows indicate the relative magnitudes of the fluid velocity at various points across the pipe. The velocity distribution curve in a circular pipe is a parabola, and the average velocity is half the maximum at the center of the pipe.

6.93. One characteristic of laminar flow (in the x direction) is that it obeys Newton's equation,*

$$F = \mu A \frac{du}{dy}, \tag{6.32}$$

where F is the shearing force (or fluid friction) over an area A between two parallel layers of fluid flowing with different velocities u in a region where the velocity gradient perpendicular to the flow direction is du/dy; the symbol μ represents the *absolute* (or *dynamic*) *viscosity* of the fluid and is defined by equation (6.32). The dimensions of viscosity are seen to be (mass)/(length)(time), i.e., kg/m·s (or Pa·s) in SI units. In the cgs system, the unit of viscosity, expressed in g/(cm)(sec), is called the poise, and it is in terms of this unit (or its hundredth part, called a centipoise) that viscosity values have frequently been calculated in the past.

6.94. Laminar flow may be distinguished from other types of flow by means of a dimensionless quantity called the *Reynolds number* (or *modulus*); this is represented by Re and defined by

$$\mathrm{Re} \equiv \frac{Du\rho}{\mu}, \tag{6.33}$$

*Certain fluids, e.g., suspensions, which do not obey the Newton equation, are referred to as non-Newtonian fluids.

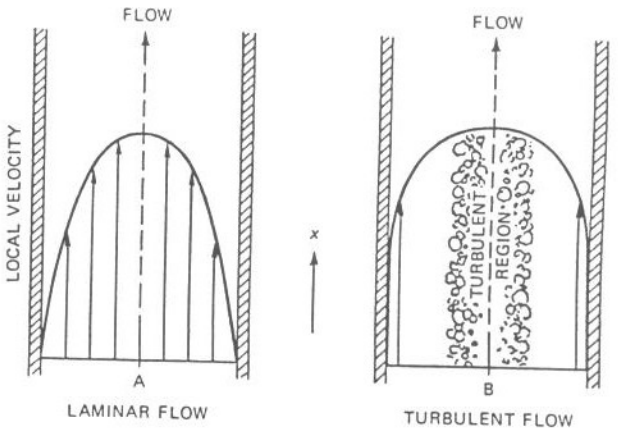


Fig. 6.11. A. Laminar flow. B. Turbulent flow.

where D is the pipe diameter, u is the mean velocity of the fluid, ρ is its density, and μ is its viscosity. Experiments with many fluids have shown that, in general, flow is laminar, and Newton's equation is obeyed in ducts of uniform cross section as long as Re is less than about 2100. The precise critical value of the Reynolds number depends, to some extent, on the flow conditions.

6.95. If the circumstances are such that Re exceeds about 4000 in such systems, the fluid motion is turbulent; this type of flow is characterized by the presence of numerous eddies which cause a radial motion of the fluid, i.e., motion across the stream, in addition to the flow parallel to the pipe axis. Newton's equation is then no longer valid.

6.96. The velocity profile for turbulent flow is shown in Fig. 6.11B. As indicated, three more-or-less distinct regions exist in a turbulent stream. First, there is a layer near the wall in which the flow is essentially laminar. This is followed by a transition (or buffer) zone in which some turbulence exists, and finally, around the pipe axis, there is the fully turbulent core. In the latter region, turbulence mixes the fluid so that the velocity in the axial direction changes less rapidly with radial distance than it does under laminar flow conditions. As a result, there is a marked flattening of the velocity profile; this flattening becomes more pronounced with increasing Reynolds number. The ratio of the mean flow velocity to the maximum ranges from about 0.75 to 0.81, as Re increases from 5000 to 100,000.

6.97. In the range between the Re values of 2100 and 4000, there is generally a transition from purely laminar flow to complete turbulence. Laminar behavior sometimes persists in this region or turbulence may increase, depending, among other factors, on entrance conditions, the occurrence of upstream turbulence, the roughness of the pipe, the presence of obstacles, and on pulsations in flow rate produced by the pump or some other component in the system.

6.98. The Reynolds number, as given by equation (6.33), is applicable only to pipes of circular cross section; for noncircular channels, such as the flow regions between fuel rods, long rectangular ducts, and annular spaces, a good approximation is obtained if D in equation (6.33) is replaced by the *equivalent* (or *hydraulic*) *diameter* D_e, defined by

$$D_e = 4 \times \frac{\text{Cross section of stream}}{\text{Wetted perimeter of duct}}, \tag{6.34}$$

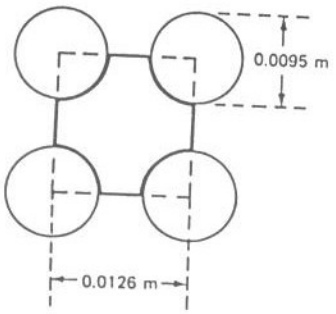
which becomes identical with the actual diameter for a circular pipe. The use of the equivalent diameter makes possible the application of relationships for circular channels to the prediction of heat-transfer coefficients, pressure drops, and burnout heat fluxes for noncircular channels, as will be seen in due course.

Example 6.5. Calculate the average Reynolds number in a PWR from the following data:

Average coolant temperature	311°C
Total coolant mass-flow rate	1.83×10^4 kg/s
Fuel rods, outer diameter	9.50 mm
pitch (square)	12.6 mm
Rod array in assembly	17 × 17
Number of assemblies	193

The coolant *mass-flow rate* (in kg/s) is equal to the *fluid velocity* u (in m/s) multiplied by the fluid density (in kg/m^3) and the flow area (in m^2).

To determine D_e, consider a unit cell containing quadrants of four adjacent fuel rods, i.e., the cell is effectively associated with a single rod. The stream cross section and wetted perimeter of the coolant flow channel per rod may then be obtained from the accompanying illustration. Thus,



$$\text{Stream cross section} = (0.0126)^2 - \tfrac{1}{4}\pi(0.0095)^2 = 8.79 \times 10^{-5}\ \text{m}^2.$$

$$\text{Wetted perimeter} = \pi(0.0095) = 0.0298\ \text{m}.$$

Hence,

$$D_e = \frac{(4)(8.79 \times 10^{-5})}{0.0298} = 1.18 \times 10^{-2}\ \text{m}.$$

Next, it is useful to determine the *mass velocity* G, which is equal to $u\rho$; this is obtained by dividing the given mass-flow rate by the corresponding flow area (or stream cross section multiplied by the total number of fuel rods*). Hence,

$$G = \frac{1.83 \times 10^4}{(17)(17)(193)(8.79 \times 10^{-5})} = 3730\ \text{kg/m}^2\cdot\text{s}.$$

Then, taking the viscosity of water at 311°C to be 8.8×10^{-5} Pa·s (kg/m·s) as given in the Appendix, it follows from equation (6.33) that

$$\text{Re} = \frac{D_e u\rho}{\mu} = \frac{D_e G}{\mu} = \frac{(1.18 \times 10^{-2})(3730)}{8.8 \times 10^{-5}} = 500{,}000.$$

6.99. Although the use of the equivalent diameter is convenient for preliminary design calculations, certain limitations should be recognized. The theoretical validity of the equivalent diameter concept for generalized use has, in fact, been questioned. For example, for a duct having a cross section in the shape of an isosceles triangle, the turbulent flow friction for air was found to be 20 percent lower than the value calculated using the equivalent diameter [4]. Heat-transfer coefficients also varied considerably around the perimeter. Several correlation procedures have been proposed for ducts with noncircular cross sections, but additional experimental work is required for their verification.

HEAT-TRANSFER COEFFICIENTS OF ORDINARY FLUIDS

6.100. In essentially all cases of reactor cooling where forced convection is used, the coolant is under turbulent flow conditions. Satisfactory predictions of heat-transfer coefficients in long, straight channels of uniform cross section can be made on the assumption that the only variables involved are the mean velocity of the fluid coolant, the diameter (or equivalent diameter) of the coolant channel, and the density, heat capacity, viscosity, and thermal conductivity of the coolant. From the fundamental differential equations or by the use of the methods of dimensional analysis,† it can be shown that heat-transfer coefficients for turbulent flow conditions can be expressed in terms of three dimensionless moduli; one of these is the Reynolds number, already defined, and the others are the *Nusselt number* (Nu) and the *Prandtl number* (Pr), defined by

$$\text{Nu} \equiv \frac{hD}{k} \quad \text{and} \quad \text{Pr} \equiv \frac{c_p \mu}{k}.$$

6.101. As a result of numerous experimental studies of heat transfer, various expressions relating the three moduli have been proposed; one of these, for an ordinary

*A small flow at the periphery of the core fuel assemblies is neglected here.
†See standard texts on heat transfer, e.g., General References for this chapter.

(nonmetal) fluid in a long, straight channel, is the Dittus-Boelter correlation [5],

$$\frac{hD}{k} = 0.023 \left(\frac{Du\rho}{\mu}\right)^{0.8} \left(\frac{c_p \mu}{k}\right)^{0.4} \tag{6.35}$$

or

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}, \tag{6.36}$$

with all the physical properties evaluated at the bulk temperature of the fluid. In a modified form [6],

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\mathrm{Pr}^{0.33},$$

the physical properties, except the specific heat, are the values at the film temperature, i.e., in the laminar (film) layer of fluid adjacent to the surface. This is taken as the arithmetic mean of the wall (or surface) temperature and the bulk fluid temperature.

Example 6.6. Calculate the heat-transfer coefficient for the water coolant in Example 6.5.

From the data in the Appendix, k at 311°C is estimated to be 0.518 W/m·K and Pr about 1.06. Hence, from equation (6.36),

$$h = \frac{(0.023)(0.518)(500{,}000)^{0.8}(1.06)^{0.4}}{0.0118} = 37{,}500\ \mathrm{W/m^2 \cdot K}.$$

6.102. It is seen from equation (6.35) that if the viscosity, thermal conductivity, density, and specific heat of the coolant are known, the heat-transfer coefficient can be estimated for turbulent flow of given velocity in a pipe or channel of specified diameter (or equivalent diameter). The results appear to be satisfactory for values of Re in excess of about 10,000, and for Pr values of from 0.7 to 120. This range of Prandtl numbers includes gases and essentially all liquids, except liquid metals; the latter have very low Prandtl numbers, primarily because of their high thermal conductivity but also often because of their low viscosity and heat capacity. The correlations for liquid metals will, therefore, be considered separately (§6.106 et seq.).

6.103. Where large differences exist between the temperatures of the solid and of the fluid, the associated variations in the physical properties of the coolant can influence the heat transfer. As indicated, such variation is most significant for the viscosity, and in order to make allowance for it the relationship

$$\mathrm{Nu} = 0.027\ \mathrm{Re}^{0.8}\mathrm{Pr}^{0.33} \left(\frac{\mu}{\mu_w}\right)^{0.14}$$

has been proposed, where μ is the viscosity at the bulk fluid temperature, and μ_w is that at the wall temperature. This equation, used in conjunction with the equivalent diameter concept, has been found to be satisfactory for predicting local and average heat-transfer coefficients for ordinary fluids flowing through thin, rectangular channels [7].

HEAT-TRANSFER COEFFICIENTS OF GASES

6.104. Heat transfer to gases is treated in much the same way as for ordinary fluids such as water, and convection heat-transfer coefficients can be predicted by means of the correlation in equation (6.36). For many gases, including helium, carbon dioxide, and air, Pr is approximately 0.70 and so equation (6.36) reduces to

$$\mathrm{Nu} \approx 0.020\ \mathrm{Re}^{0.8}.$$

The marked differences in physical (especially thermal) properties result, however, in some important general differences in heat-transfer behavior between gases and liquids. For example, the thermal conductivity of helium gas at atmospheric pressure is only about a third that of water although it is enhanced by an increase in pressure. In considering the transport of heat away from the heated surface by the fluid, the specific heat of the latter is important. Here again, an increase in the gas pressure is advantageous; there is a density increase accompanied by a corresponding increase in the heat capacity per unit volume.

6.105. In so-called high-speed flow, where the gas velocity is greater than about 0.2 that of sound (§6.158), certain special effects occur that influence the temperature driving force for convection heat transfer. For example, "aerodynamic heating" results from the frictional effects in the boundary layer, and kinetic energy considerations (stagnation effects) lead to a difference between flowing and "at rest" conditions. For such matters, heat-transfer texts should be consulted [8].

HEAT TRANSFER TO LIQUID METALS

INTRODUCTION

6.106. For ordinary fluids, the principal mechanism of heat transport is by the effect of turbulence, as a result of which a "parcel" of fluid is rapidly moved from a region close to the hot wall into the main body of fluid. In liquid metals, however, thermal transport occurs mainly by molecular conduction. Whereas this mechanism may provide 70 percent of the heat transfer for a liquid metal, it contributes only about 0.2 percent to heat transfer in water. This means that the laminar boundary thickness, which is important for ordinary liquids, is not significant for liquid metals, and heat-transfer relationships applicable to gases and nonmetallic liquids cannot be used.

6.107. The essential difference between the heat-transfer properties of liquid metals and ordinary fluids is illustrated by the temperature profiles in a heated tube shown in Fig. 6.12 [9]. In these curves, the approach of the fluid temperature to the tube-wall temperature is represented by the dimensionless quantity $(t_w - t)/(t_w - t_0)$, where t_w and t_0 are the wall and centerline temperatures, respectively. The abscissa is the ratio y/r_0, where y is the distance from the wall at which the fluid temperature

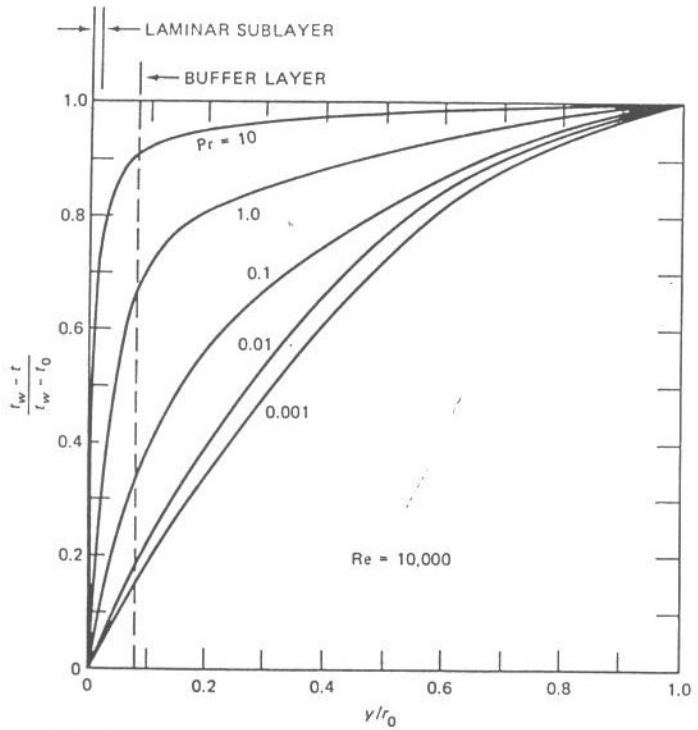


Fig. 6.12. Dependence of temperature profile on Prandtl number [9].

is t, and r_0 is the tube radius. The Prandtl numbers are the parameters for the various curves, and the Reynolds number is 10^4 in all cases.

6.108. For Pr = 1, the velocity and temperature profiles are identical; most of the resistance to heat transfer occurs in the laminar sublayer and in the buffer layer, and there is little further change in temperature as the center of the tube ($y/r_0 = 1$) is approached. For liquid metals ($\mathrm{Pr} \ll 1$), however, molecular conduction is so significant that there is a marked thermal gradient from the buffer layer boundary all the way to the center, much as would be observed for a solid rod (Pr = 0). The heat-transfer coefficient is normally based on a mixed-mean temperature obtained by integration of the thermal profile (§6.52). It can be seen that for fluids of low Prandtl number this temperature may be quite different from that at the centerline.

HEAT-TRANSFER COEFFICIENTS OF LIQUID METALS

6.109. According to the Reynolds analogy [10], the transfer of heat and momentum between a solid wall and a fluid in turbulent motion occur by similar mecha-

nisms. This analogy has led to the following correlation for liquids of high molecular conductivity flowing in long, cylindrical pipes [11]:

$$\frac{hD}{k} = 7 + 0.025 \left(\frac{Du\rho c_p}{k}\right)^{0.8}$$

or

$$\mathrm{Nu} = 7 + 0.025\ \mathrm{Pe}^{0.8}, \tag{6.37}$$

where Pe is the *Peclet number*, defined by

$$\mathrm{Pe} \equiv \mathrm{Re} \times \mathrm{Pr} \equiv \frac{Du\rho c_p}{k}.$$

Although equation (6.37) has been used quite extensively for liquid metals, many designers prefer the more conservative relationship [12]

$$\mathrm{Nu} = 0.625\ \mathrm{Pe}^{0.4}. \tag{6.38}$$

6.110. When conduction is the predominant heat-transfer mechanism, as is the case for liquid metals, the equivalent-diameter concept for noncircular channels (§6.98) is not strictly applicable. However, since more accurate correlations are too complex for general use, a common practice is to calculate Nu and Pe for liquid metals in the same way as for nonmetallic fluids. A modified expression, which is the result of multiplying the right side of equation (6.37) by 0.8, namely,

$$\mathrm{Nu} = 5.6 + 0.020\ \mathrm{Pe}^{0.8},$$

has been proposed for flow in the channel between parallel plates with heat being removed uniformly from one side only [13]. The equivalent diameter used in computing Nu and Pe is obtained from equation (6.34) as twice the distance between the plates.

EDDY CONDUCTIVITY AND HEAT TRANSFER

6.111. Another approach to the prediction of heat-transfer coefficients is based on the use of the eddy conductivity. Although the concept is quite general, it is more useful (as will be seen shortly) for the description of heat transfer to liquid metals than to other (normal) liquids. Turbulent flow can be treated as the sum of a molecular effect and a turbulent effect. For laminar (or molecular) flow, the shear (or drag) force in a viscous fluid is expressed by equation (6.32), and for the present purpose it is convenient to write this equation in the form

$$\frac{F}{A} = \tau = \mu \frac{du}{dy}, \tag{6.39}$$

If h_b represents the boiling heat-transfer coefficient, it may be defined by

$$\frac{q}{A} = h_b \Delta t_b,$$

so that

$$h_b = C(\Delta t_b)^{n-1}.$$

6.138. For subcooled or local boiling, the relationship

$$\frac{q}{A} = \left(\frac{e^{p/6.2}}{0.79}\Delta t_b\right)^4 \tag{6.46}$$

has been found to be applicable, where q/A is in W/m^2, p is the pressure in MPa, and Δt_b is in kelvins [20]. In the region of a PWR channel prior to the inception of local boiling, the heat-transfer coefficient is predicted using equation (6.35). Since equation (6.46) applies to the local-boiling region, designers often define the initiation of local boiling as the point where the surface temperatures predicted by the two equations become equal.

6.139. Although the heat-transfer coefficient is of little importance in the design of a BWR, it is often desirable to evaluate the surface temperature of the fuel elements corresponding to a desired average heat flux. This temperature may be important from the standpoint of corrosion resistance, and it may also serve as a reference for estimating the maximum internal temperature within the fuel element. According to equation (6.46), the temperature difference Δt_b at the surface is proportional to the 0.25 power of the heat flux. The surface temperature itself would thus appear to be relatively insensitive to changes in the flux.

Example 6.9. Determine the surface temperature of the fuel in a reactor core under subcooled-boiling conditions at a system pressure of 7.2 MPa when the average heat flux is (*a*) 0.5 MW/m^2 and (*b*) 5 MW/m^2. The saturation temperature of water at this pressure is 288°C.

(*a*) From equation (6.46),

$$\Delta t_b = \frac{0.79}{e^{p/6.2}}\left(\frac{q}{A}\right)^{0.25}$$

$$= \frac{0.79}{e^{7.2/6.2}}(5 \times 10^5)^{0.25} = 6.6 \text{ K } (6.6^\circ\text{C})$$

The fuel surface temperature is consequently 288 + 6.6 ≈ 295°C.

(*b*) For an average heat flux of 5.0 MW/m^2,

$$\Delta t_b = (6.6)(10)^{0.25} = 12 \text{ K } (12^\circ\text{C}),$$

and so the fuel surface temperature is 288 + 12 = 300°C. (As stated, the surface temperature is not very sensitive to changes in the heat flux; the temperature increases

from 295°C to only 300°C for a tenfold increase in heat flux. However, the higher flux may be above the critical heat flux limit.)

CORE HYDRAULICS

PRESSURE DROP IN LAMINAR FLOW

6.140. Pumping-power requirements are determined by the pressure drop in the cooling system and the rate of flow of the coolant. The total pressure drop is the sum of terms arising from (1) the hydrostatic pressure due to the increase in level, (2) the acceleration of the coolant fluid, and (3) friction in channels, pipes, etc., and in regions where there are changes in the cross-sectional area. Of these, the hydrostatic pressure is simply the integrated product of the increase in level, the density, and the acceleration of gravity. The kinetic energy term, for a given mass-flow rate, depends on the difference in velocity between inlet and outlet. For an incompressible fluid this is usually negligible, and even for gases the contribution is very small for reactor operating conditions. The subsequent discussion will therefore be restricted to the effects of friction and flow area changes on the pressure drop. Although these topics are treated in standard texts on fluid dynamics, they will be reviewed briefly here.

6.141. The pressure drop Δp_f accompanying isothermal *laminar* flow at a mean velocity u in a cylindrical pipe of diameter D and length L is given by the familiar Poiseuille equation

$$\Delta p_f = \frac{32\mu u L}{D^2},$$

which may be written as

$$\Delta p_f = 64\,\frac{\mu}{Du\rho}\cdot\frac{L}{D}\cdot\frac{\rho u^2}{2}, \tag{6.47}$$

where μ is the viscosity.* The subscript f is used in p_f to indicate that the pressure loss described is due to fluid friction. The factor $\mu/Du\rho$ on the right of equation (6.47) is the reciprocal of the Reynolds number, and so

$$\Delta p_f = \frac{64}{\text{Re}}\cdot\frac{L}{D}\cdot\frac{\rho u^2}{2}. \tag{6.48}$$

This gives the pressure drop due to fluid friction for laminar flow in a straight pipe of circular cross section. It should be noted that replacement of D by the equivalent diameter D_e, as defined in §6.98, is not valid for laminar flow in channels which do

*If English units are used in equation (6.47), or in subsequent equations, g_c, the factor relating force and mass, must be used as appropriate.

not have circular cross sections, although it is a good approximation for turbulent flow (§6.144).

PRESSURE DROP IN TURBULENT FLOW

6.142. For turbulent flow at a mean velocity u, the relationship corresponding to equation (6.48) is

$$\Delta p_f = 4f\frac{L}{D}\cdot\frac{\rho u^2}{2}, \tag{6.49}$$

known as the Fanning equation, in which the dimensionless quantity f is called the *friction factor.** Comparison of equations (6.48) and (6.49) shows that for laminar flow $f = 16/\mathrm{Re}$, but the relationship between the friction factor and the Reynolds number in the case of turbulent flow is more complicated. Several empirical expressions have been developed; one of the simplest of these, which holds with a fair degree of accuracy for flow in smooth pipes at Reynolds numbers up to about 2×10^5, is the Blasius equation

$$f = 0.079\ \mathrm{Re}^{-0.25}. \tag{6.50}$$

This permits the evaluation of the turbulent-flow friction factor for a given Reynolds number.

6.143. For turbulent flow in a commercial rough pipe, the friction factor is larger than for a smooth pipe at the same Reynolds number, as shown in Fig. 6.17. The deviation from the ideal behavior of a smooth pipe increases with the roughness of the pipe, especially at high Re values. The variation of the friction factor with Reynolds number has been determined experimentally for different degrees of roughness, expressed in terms of a dimensionless quantity ϵ/D, where ϵ is a measure of the size of the roughness projections and D is the pipe diameter [21].

6.144. For turbulent flow in noncircular channels of relatively simple form, the pressure drop due to friction may be calculated by substituting the equivalent diameter for D in equation (6.49). The same value is used in determining the Reynolds number. For in-line flow along channels in rod bundles, the friction factor depends to some extent on the pitch-to-diameter ratio. For initial design purposes only, however, a correction factor of 1.3 is a reasonable approximation for the usual range of these ratios [22].

Example 6.10. Estimate the pressure drop required to overcome friction for the turbulent flow of water in the channel between the fuel rods in Example 6.5, along a length of 4.17 m.

The value of Re was found to be 5.00×10^5; hence, assuming the fuel rods to be moderately smooth, the value of f is seen from Fig. 6.17 to be about 0.0032.

The mass velocity $G(=u\rho)$ in Example 6.5 was 3730 kg/m^2·s, and since ρ for water at 311°C is 0.691×10^3 kg/m^3, u is 3730/691 = 5.40 m/s. Hence, with D_e equal to

*In the equivalent Darcy-Weisbach equation a friction factor equal to $4f$ is used.

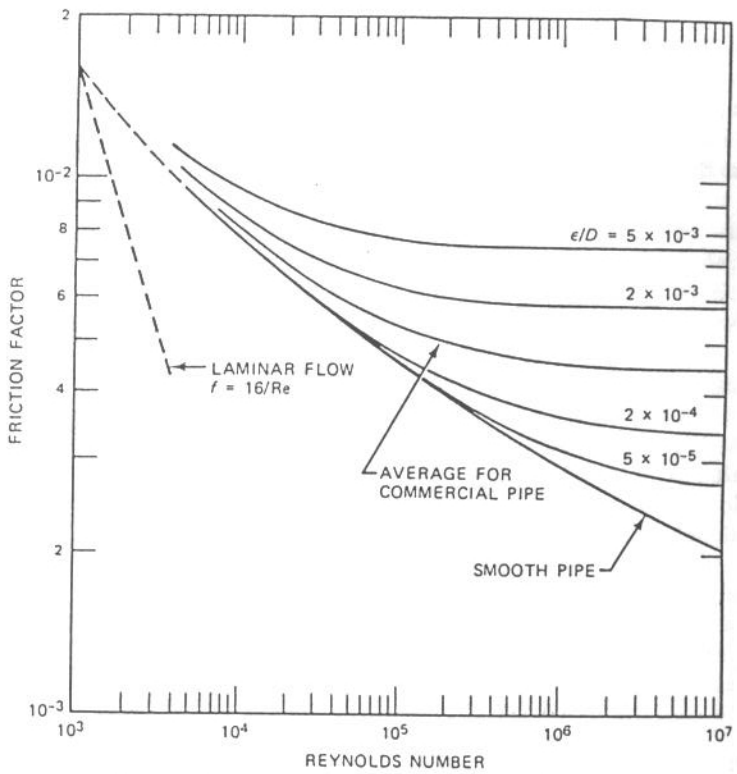


Fig. 6.17. Friction factor as a function of pipe roughness [21].

0.0118 m, it follows from equation (6.49) that

$$\Delta p_f = (4)(0.0032)\,\frac{4.17}{0.0118}\cdot\frac{(691)(5.40)^2}{2} = 4.56 \times 10^4\ \mathrm{Pa}.$$

Upon applying the 1.3 correction for flow in rod bundles (§6.144), the result is

$$\Delta p_f = 4.56 \times 10^4 \times 1.3 \approx 6 \times 10^4\ \mathrm{Pa}.$$

This calculation does not take into account the pressure losses in the flow channel resulting from the spacer grids (normally about six) provided at intervals to support the rods and enhance mixing (§6.148).

VELOCITY HEAD LOSSES

6.145. In a flowing fluid there will be changes in pressure due to changes in velocity resulting from gradual or abrupt changes in flow area. Such pressure changes are usually considered in terms of the *velocity head*, defined as $u^2/2$; the pressure cor-

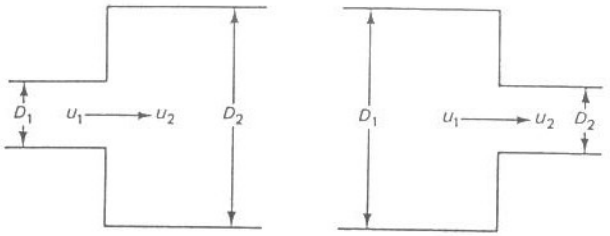


Fig. 6.18. A. Abrupt expansion in fluid flow. B. Abrupt contraction in fluid flow.

responding to a head H is essentially equal to $H\rho$.* Since u^2 is generally high for turbulent flow, the pressure losses accompanying changes in cross-sectional area of fluid conduits may be very significant. In general, for abrupt expansion (Fig. 6.18A) or contraction (Fig. 6.18B), the pressure change Δp due to the loss of head ΔH, which occurs in either case, can be represented by

$$\Delta p = \rho \Delta H(\text{expansion}) = K_e \frac{\rho u_1^2}{2}$$

$$\Delta p = \rho \Delta H(\text{contraction}) = K_c \frac{\rho u_2^2}{2},$$

where u_1 and u_2 are the upstream and downstream velocities, respectively; the value of K, the loss coefficient, depends upon the conditions.

6.146. For abrupt *expansion*,

$$K_e = \left(1 - \frac{D_1^2}{D_2^2}\right)^2,$$

where D_1 and D_2 are the pipe diameter upstream and downstream, respectively. For expansion into a large reservoir, D_2 is large, and K_e becomes virtually unity. The loss of head is then almost equal to $u_1^2/2$.

6.147. For abrupt *contraction*, the value of K_c varies with the ratio D_2/D_1 in the following manner:

D_2/D_1	0.8	0.6	0.4	0.2	0
K_c	0.13	0.28	0.38	0.45	0.50

For the case of inlet from a very large reservoir, D_2/D_1 approaches zero, and K_c is then approximately 0.5; the corresponding loss of head is $u_2^2/4$. It should be understood that these results apply only to cases of abrupt expansion or contraction. The loss of head decreases if the fluid exit is rounded, making the change less abrupt. Where the exit is tapered so that the included angle is 7° or less, the losses will usually be negligible.

*Since the velocity head is equal to the kinetic energy of a unit mass of flowing fluid, the term arises from the similarity with the potential energy of a height (or "head") of a unit fluid mass.

Example 6.11. Estimate the pressure loss due to contraction and expansion of coolant as it enters and then leaves a channel between the fuel rods considered in Example 6.10.

The coolant undergoes sudden contraction and expansion as it enters and leaves the channel, respectively, from or to a header or manifold of large cross section. The loss of head upon entering the channel, i.e., upon contraction, may then be taken as $0.5u_2^2/2$, where u_2 is the velocity of the coolant in the channel. Similarly, upon leaving the channel, the loss is $u_1^2/2$, where u_1 is also the velocity of the coolant in the channel. The total loss of head upon entry and exit is thus $1.5u^2/2$, where u is the velocity of the coolant in the channel, i.e., 5.40 m/s (Example 6.10). The pressure loss is equal to the loss of head multiplied by the density of the coolant; hence,

$$\text{Pressure loss} = \frac{(1.5)(5.40)^2(691)}{2} = 1.5 \times 10^4 \text{ Pa}.$$

6.148. Additional contraction and expansion losses are introduced in the flow channels between long fuel rods by spacer grid assemblies which support the fuel rods at intervals along their length. Loss coefficients for such a special geometry can best be determined by experiment. However, for an order of magnitude estimate of the effect, a loss of one velocity head for each spacer grid may be assumed to account for both contraction and expansion. The pressure loss is thus roughly $\rho u^2/2$ per grid. In Examples 6.10 and 6.11, the loss for six grids would be about 6×10^4 Pa.

6.149. Losses in pipe fittings, due to changes in direction, e.g., in elbows, curves, etc., or to contraction in valves, can also be expressed in terms of the velocity head, e.g., $K_1 u^2/2$. The factor K_1 may range from 0.25 or less for a gradual 90° curve or a fully opened gate valve, to 1.0 for a standard screwed 90° elbow, or as high as 10 for a fully opened globe valve.*

TWO-PHASE FLOW

6.150. When a liquid reactor coolant vaporizes in a channel, the resulting cocurrent flow of liquid and vapor has special characteristics which are important in both steady-state and transient hydraulic analysis. The resistance to flow is particularly significant in considering fuel-rod cooling under postulated accident conditons where normal flow may be reduced or emergency core coolants are being injected (§11.81). With the heat addition resulting in vaporization, the channel flow becomes complex

*For further information, standard reference books and manufacturers' catalogs should be consulted. The loss coefficients K_1 are often expressed in terms of an "equivalent" length of straight pipe.

since the fraction of vapor increases from point to point along the flow path with resulting changes in hydrodynamics which, in turn, can affect the vaporization rate. For orientation purposes, only some of the general characteristics of two-phase flow are considered here.

6.151. An important feature of two-phase flow is the distribution of the phases into a number of characteristic flow patterns dependent upon the mass velocity of each phase, orientation (whether horizontal or vertical), and fluid properties. The various patterns are of three basic types:

Bubbly Flow: At low void fractions, bubbles of vapor are distributed in a continuous liquid phase.

Slug Flow: At relatively low flow velocities and moderate void fractions, the vapor bubbles may agglomerate and form large slugs in the channel of liquid and bubbles which can lead to instabilities.

Annular Flow: Here, there is a vapor core in the channel within a liquid phase annulus along the wall. There are some vapor bubbles within the liquid annulus, and liquid droplets may be suspended in the vapor core. Annular flow is expected at high vapor fractions and high flow velocities. In subcooled stable film boiling, a reverse annular flow occurs with the vapor film along the wall and the liquid continuous in a central core.

TWO-PHASE PRESSURE DROP

6.152. Two basic approaches have been used for engineering-design, pressure-drop calculations. In *homogeneous* models, the two phases are assumed to flow as a single phase possessing mean flow properties and a suitable single-phase friction factor is developed to represent the two-phase flow. *Separated-flow* models consider the phases to be segregated into two separate streams, with each moving at a different velocity. Empirical correlations are then used to determine a so-called friction multiplier (described in the next paragraph) which is a function of flow parameters. The different models vary in sophistication and may be specific to given flow regimes. Furthermore, models must be used with care since the behavior being modeled is inherently complex [23] with numerous simplifications necessary for a practical description.

Boiling-Water Reactors

6.153. The problem of the two-phase pressure drop in a vertical channel is of particular importance in BWR design. One treatment involves the use of a *friction-factor multiplier* R, defined as the ratio of the two-phase friction-pressure gradient $(dp/dL)_2$ to the single-phase value for the liquid phase alone $(dp/dL)_l$, i.e.,

$$R \equiv \frac{(dp/dL)_2}{(dp/dL)_l} \approx \frac{(\Delta p)_2}{(\Delta p)_l},$$

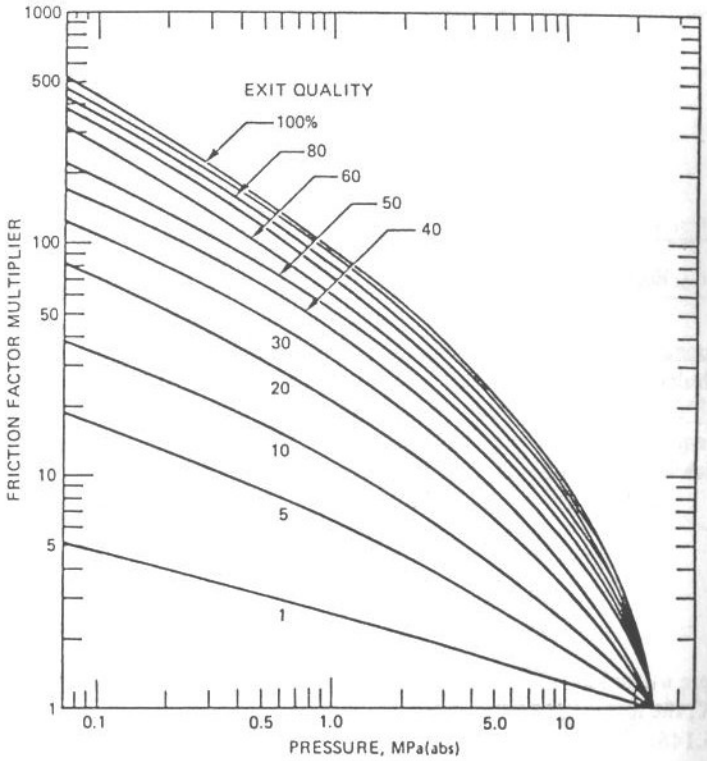


Fig. 6.19. Friction-factor multiplier for two-phase flow [24].

where $(\Delta p)_2$ is the two-phase pressure drop in a given channel in which $(\Delta p)_l$ would be the pressure drop for the liquid phase alone. Values of R, obtained by semiempirical correlation procedures and graphical integration of local values with respect to length, are given in Fig. 6.19 as a function of the system pressure for steam ranging in exit quality from 1 to 100 percent [24]. This figure is applicable when the axial heat flux distribution is uniform; other curves have been determined for the situation in which the axial heat flux has a sinusoidal distribution.

Pressurized-Water Reactors

6.154. In the design of the first PWRs, local (nucleate) boiling was considered to be undesirable, and conditions were chosen to prevent its occurrence. In later designs, however, local boiling during steady-state operation became a design requirement. Local boiling will affect the pressure drop and, in an open lattice, will result

would be undesirable because of the change in density of the fuel and the release of fission-product gases, among other effects.

6.188. A principle that has been useful in reactor design is to relate the nominal (or average) performance to the maximum value that can be expected anywhere in the core. To this end, each of several relevant design specifications is assigned an adjustment or correction factor which represents the ratio of the maximum to average values of such parameters as heat flux, coolant-flow rate, and enthalpy rise that would result from the most probable variations in the given specifications. The various factors are then combined in a suitable manner to yield the overall ratio of the maximum to average values of a particular operating parameter. The constraints mentioned must then apply to the maxima determined in this manner.

HOT-CHANNEL FACTORS

6.189. Systematic approaches involving the various adjustment factors have not been standardized in reactor design; hence, the following discussion is intended only to illustrate the general nature of the procedures employed. The "hot-channel" concept commonly used is based on the assumption that a reactor core, with solid fuel elements having coolant passages (or channels) between them, will have one channel in which a combination of heat-generation rate per unit volume (or power density) and variations in dimensions produce a heat flux that is greater than at any other point in the core. Such a hypothetical channel is called the *hot channel* and a design requirement is that the conditions is this channel shall lie well within the postulated limiting operating conditions.

6.190. Various hot-channel factors are used to compare the expected maximum values of certain operating parameters, especially the heat flux and the enthalpy rise, in the hot channel with the average values in the core as a whole. Since the heat flux depends on the neutron flux, which varies with position in the core, the hot-channel factor for the heat flux is, in principle, associated with a particular location in the hot channel. The hot-channel factor then refers to the "hottest" spot in the channel; it is sometimes called a *peaking factor* because the heat flux at this spot is higher than anywhere else in the core. The enthalpy rise, on the other hand, depends on the temperature rise of the coolant which increases continuously as it flows through a channel; the hot-channel factor is then characteristic of the channel and not of a particular location in the channel.

6.191. Hot-channel factors (and subfactors) fall generally into two main categories, namely, nuclear factors and engineering factors. Nuclear factors are essentially those related to the neutronic aspects of core design, whereas engineering factors arise largely from fabrication variations and some complexities in the flow of the coolant. Factors of these two general types will be considered separately in the following sections.

6.192. Nuclear factors are concerned with design specifications that affect the local heat-generation rate per unit volume. This quantity is determined by the local fission rate which depends on the neutron flux and its energy spectrum, the microscopic fission cross sections of the fissile material at the various neutron energies, and the isotopic composition of the fuel. Apart from the variations in other quantities, the neutron flux will vary in both axial and radial directions in the core. For a bare, uniformly loaded core, a sinusoidal distribution in the flux is to be expected (§6.176), but this distribution will be disturbed by the reflector, by local perturbations produced by control elements, and by nonuniform loading. Nevertheless, there is usually a location near the center of the core where the fission rate (and heat flux) is maximal. In general, the spatial dependence of the fission rate can be determined from neutronic calculations, and hence a power peaking factor can be evaluated which is primarily related to the core nuclear design specifications. This peaking factor is then assumed to apply to the hot channel.

Engineering Hot-Channel Factors

6.193. Engineering factors are concerned primarily with possible variations and uncertainties in core component dimensions and in structural features that are likely to affect the heat flux and the coolant flow. The hot-channel engineering factors are obtained by combining a number of subfactors in an appropriate manner. Some of these subfactors lend themselves to calculation based on manufacturing quality-control data, but others must be estimated in different ways.

6.194. A simple example of an engineering subfactor is one related to variations in the spacing (or pitch) of the fuel elements. In this case, it is possible from the manufacturing tolerances on the fuel-rod assemblies to determine the associated coolant-flow variation. Where the coolant channels are in parallel, a spacing that is less than average in a particular channel will lead to a lower flow in that channel. This lower flow rate will affect both the temperature rise (and enthalpy rise) of the coolant and the temperature difference between the coolant and the fuel-rod surface.

6.195. For turbulent flow, with the pressure drop Δp_f being principally due to drag friction, equation (6.49) gives

$$\Delta p_f = 4f \frac{L}{D_e} \cdot \frac{\rho u^2}{2}, \tag{6.70}$$

where the various quantities have been defined earlier. If the channels are in parallel, the pressure drop through the hot channel will be the same as in an average channel; consequently, from equation (6.70),

$$\frac{u_{\text{hot}}}{u_{\text{av}}} = \left(\frac{f}{D_e}\right)^{0.5}_{\text{av}} \left(\frac{D_e}{f}\right)^{0.5}_{\text{hot}}, \tag{6.71}$$

HEAT FLUX RELATED LIMITATIONS IN PRESSURIZED-WATER REACTORS

6.205. The first design limitation to be considered is related to the heat flux at which a boiling crisis could occur; for a PWR this is the DNB flux, which can be computed by means of a suitable correlation (§6.134). The DNB flux is expected to decrease as the coolant quality increases, since the larger the vapor content of the fluid, the closer it would be to the conditions at which DNB occurs.* The quality increases as the enthalpy of the fluid is increased; hence, the enthalpy rise has an important bearing on the value of the DNB flux. In computing this flux for design purposes, the maximum value of the enthalpy, i.e., the value in the hot channel, must therefore be used.

6.206. Since the enthalpy of the coolant increases as it flows upward through the core, the (computed) DNB flux decreases correspondingly, as depicted in Fig. 6.23. Assuming a sinusoidal axial distribution of the volumetric heat source, as in §6.176,

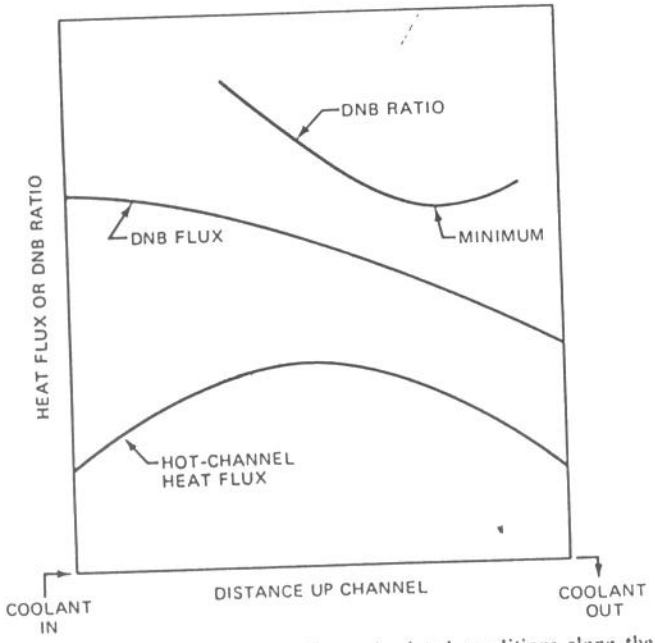


Fig. 6.23. Qualitative representation of heat flux and related conditions along the hot channel in a pressurized-water reactor. (A sinusoidal axial distribution of the heat flux is assumed for simplicity.)

*Since the average core outlet temperature in a PWR is below the saturation temperature, little or no vapor leaves the reactor vessel. However, there is substantial subcooled (local) boiling in the flow close to the heated surface, and bulk boiling may occur in the "hot channels."

the heat flux along the hypothetical hot channel will ideally have the same general distribution, as indicated in the figure. The maximum of this curve, representing the highest possible heat flux in the core for normal operating conditions, is equal to the average core heat flux multiplied by the overall heat flux hot-channel factor. In practice, the axial heat flux will not have a sinusoidal distribution, and this must be taken into consideration by the designer. For the present purpose, however, the ideal axial distribution may be assumed.

6.207. Comparison of the computed DNB heat flux with the hot-channel value at any point along the flow channel gives the *departure-from-nucleate-boiling ratio* (DNBR) or *critical heat flux ratio* (CHFR) at that point. The results for the whole channel are shown by the uppermost curve in Fig. 6.23. It is seen that the DNBR passes through a minimum. In the interest of reactor safety, an essential requirement of reactor design at present is that the minimum DNBR shall be greater than 1.3 for the hottest channel at the 112 percent overpower specified for a PWR (§6.170). This is one of the important constraints in reactor core design. It should be noted that the minimum DNBR does not occur at the point of maximum heat flux but more toward the exit where the enthalpy of the coolant is higher and the DNB flux is lower (§6.134). Factors that change the axial neutron (and heat) flux distribution, e.g., insertion of control rods, will affect the location of the minimum DNBR.

6.208. The design constraint just considered should provide safety from the onset of DNB under normal operating conditions, allowing for a certain amount of overpower that could arise from instrumental errors or minor reactivity transients. However, no allowance is made for abnormal situations that might result from excessive overpower or inadvertent decrease in the coolant-flow rate. These matters are aspects of reactor safety considered in Chapter 11.

6.209. A somewhat more sophisticated approach preferred by many PWR designers uses the *CHF power ratio* as a measure of overpower that can be tolerated before the critical heat flux (CHF) condition is reached in the hot channel. In this method, the CHF correlation curve is recalculated by computer, point by point, as the channel power levels under consideration are changed, thus reflecting shifts in mass-flow rate and other parameters. On a thermal flux, coolant enthalpy coordinate system, a family of curves are then plotted showing the proposed channel flux and corresponding variation of CHF, each at the same power level. The CHF power ratio is defined as the power at which the two curves would be tangent to one another, divided by the design or "rated" power. Since relative displacements of the curves with power level are considered, the margin is more physically meaningful than the DNBR [27].

6.210. The central fuel temperature of a cylindrical fuel rod depends on Qa^2, where a is the diameter of the fuel pellet, as may be seen from §6.67 et seq. and Example 6.3. By equation (6.20), this is related to the linear heat rate q_L, which is a characteristic property of the fuel material for the specified temperature range. Experiments with uranium dioxide, commonly used as fuel in water-cooled (and moderated) reactors, have shown that melting does not occur until the linear heat rate in the

TABLE 6.4. STATISTICAL RESULTS FOR BURNOUT IN A PRESSURIZED-WATER REACTOR AT THE 99 PERCENT CONFIDENCE LEVEL

Percent of full power	No. of possible DNB (out of 36,816 rods)	Percentage of rods protected	Minimum DNBR
	Maximum Design Condition		
100	3	99.992	1.75
112	41	99.889	1.39
	Most Probable Design Condition		
100	1	99.997	1.93
112	15	99.959	1.55

The DNB data are then evaluated for each fuel rod (or coolant channel) in the usual manner.

6.218. The results obtained from the foregoing computations are treated statistically to derive the number of possible burnouts at a specified confidence level for various operating conditions. Table 6.4 gives the values obtained at a 99 percent confidence level for both maximum and most probable design conditions of a PWR with 36,816 rods operating at 100 percent of design power and at 112 percent overpower. It is seen that, under the worst conditions that might arise in normal operation, i.e., at maximum design conditions and 112 percent overpower, there is a 99 percent confidence that almost 99.9 percent of the fuel rods will be protected from burnout. The minimum DNBR under these conditions is 1.39 at the same confidence level.

BOILING-WATER REACTORS

6.219. The design approach based on hot-channel and peaking factors used for BWRs is similar in principle to that described earlier for a PWR system, although there are differences in details. In general, engineering factors are not treated separately but are included in the peaking factors. For example, the hot-channel factor for the heat flux is taken as the product of three factors: the relative assembly power factor, the local peaking factor, and the axial peaking factor. The relative assembly power factor is the total power generated in a fuel assembly divided by the average assembly power over the whole core. The local peaking factor is the ratio of the maximum heat flux in a fuel assembly to the assembly average at the same level in the reactor core. The product of these two factors is essentially equivalent to the radial factor $F_{\Delta H}^N$ defined in §6.200. Finally, the axial factor, equivalent to F_z^N, is defined as the ratio of the maximum heat flux along the length of a fuel rod, i.e., the peak flux in the hot channel, to the average for that rod (or channel).

6.220. Typical values of the three factors for a boiling-water reactor are as follows:

Relative assembly power factor	1.40
Local peaking factor	1.13
Axial peaking factor	1.40

The overall hot-channel (peaking) factor for the heat flux is thus

$$F_q = 1.40 \times 1.13 \times 1.40 = 2.21.$$

The hot-channel factor for the enthalpy rise does not include the axial factor and it is consequently

$$F_{\Delta H} = 1.40 \times 1.13 = 1.58.$$

The average heat flux for a typical BWR is about 5×10^5 W/m^2; the maximum (hot-channel) flux is consequently $(2.2)(5 \times 10^5) = 1.1 \times 10^6$ W/m^2. The outer diameter of a clad fuel rod is 0.0125 m; the linear heat rates are, consequently, 20 kW/m average and 44 kW/m maximum.

6.221. As for a PWR, an important limiting design condition for a BWR is related to the critical heat flux (or CHF) at which a boiling crises would be expected to occur. This flux, as computed on the basis of an acceptable procedure (§6.135), is a function of the local steam quality (and hence of the enthalpy rise), the mass-flow rate of the coolant, the flow-area geometry, and the pressure. Like the DNB flux, the CHF decreases with increasing enthalpy of the coolant, and the ratio of the CHF at any point in the flow channel to the actual flux at that point, i.e., the CHF ratio (or CHFR), passes through a minimum. The design limitation is that the minimum CHFR in the hot channel must exceed 1.9 at 120 percent overpower.

6.222. In determining the most vulnerable location in a BWR core, as indicated by the minimum CHFR, consideration must be given to the change in the neutron flux shape (or power density distribution) that may result from the control elements, which enter from the bottom of the core (§1.100), and from the presence of steam voids in the coolant. Furthermore, except toward the end of the core life when the control rods are withdrawn, these rods may be used to shape the neutron flux. The accompanying spatial changes in the heat-generation rate must also be borne in mind in connection with the limitation on the linear heat rate in order to avoid central fuel melting.

6.223. A figure of merit used for BWR design and operation is the *critical power ratio* (CPR). Correlations of experimental data yield the steam quality, or "critical quality," at which the transition from nucleate boiling occurs as a function of mass-flow rate, power, and other parameters. The "critical power" is defined as that fuel bundle power which would produce the critical quality at some point within the bundle. The critical power ratio is then defined as the ratio of the critical power to the operating bundle power, at the reactor condition of interest. Design and operating limits are then established in terms of the CPR [29].