ESERCIZI SU DOMINI DI FUNZIONI DI PIÚ VARIABILI

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ESERCIZI

DETERMINARE L'INSIEME DI DEFINIZIONE DELLE SEGUENTI FUNZIONI:

1. $f(x,y) = \sqrt{x} + \sqrt{y}$

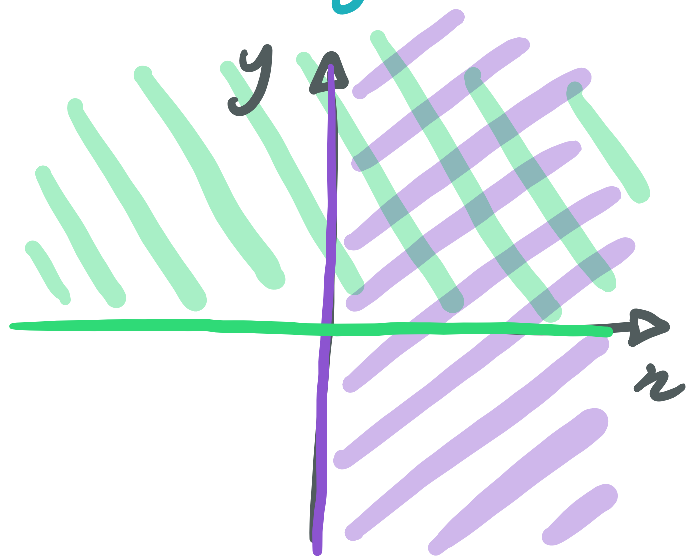
2. $f(x,y) = \sqrt{x} \cdot \sqrt{y}$

3. $f(x,y) = \sqrt{x \cdot y}$

1. $f(x,y) = \sqrt{x} + \sqrt{y}$

$\sqrt{x} \longrightarrow x \geq 0$

$\sqrt{y} \longrightarrow y \geq 0$



AFFINCHÉ f SIA DEFINITA IN (x,y), LE DUE CONDIZIONI DEVONO ESSERE ENTRAMBE SODDISFATTE

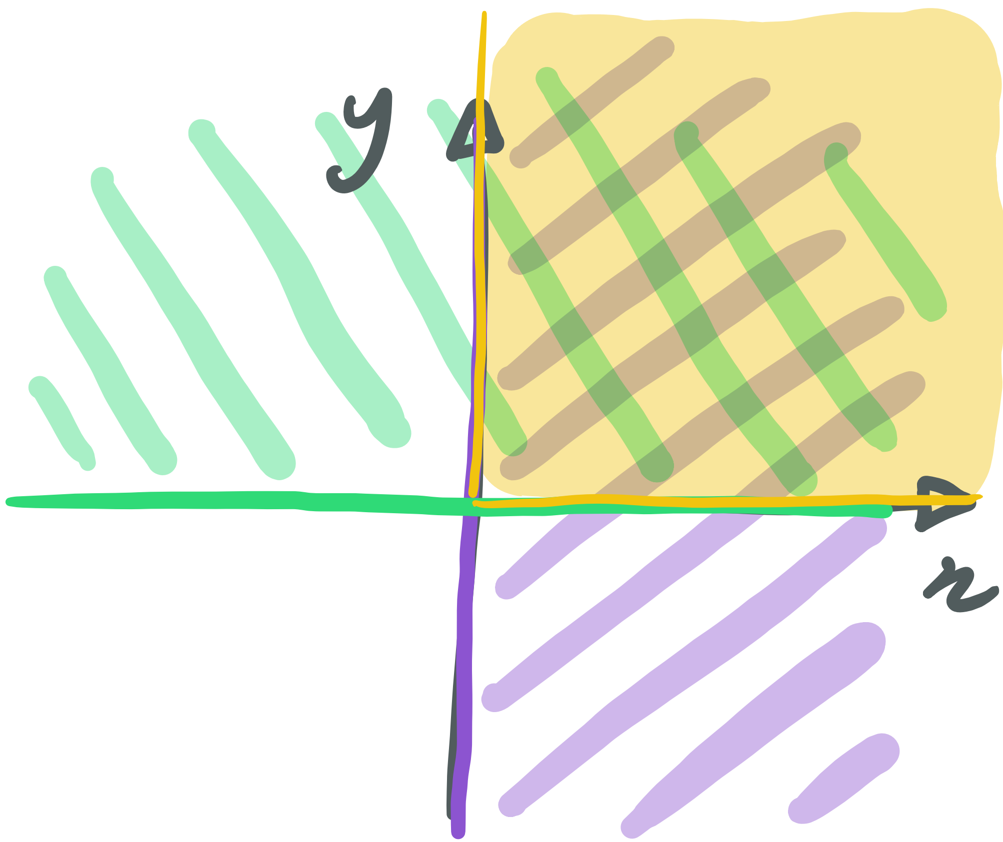
↳ CIOÈ, VANNO MESSE A SISTEMA $\begin{cases} x \geq 0 \\ y \geq 0 \end{cases}$

1. $f(x,y) = \sqrt{x} + \sqrt{y}$

QUINDI,

$$D_f = \{ (x,y) \in \mathbb{R}^2 \mid x \geq 0 \wedge y \geq 0 \}$$

CIOÈ,

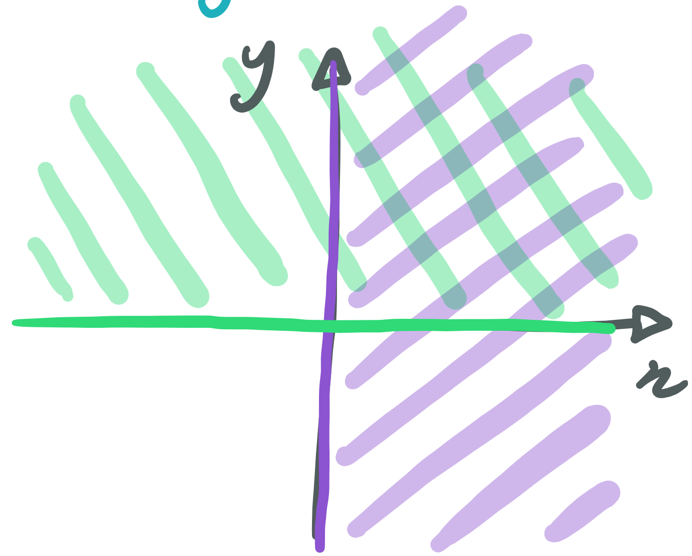


ok

2. $f(x,y) = \sqrt{x} \cdot \sqrt{y}$

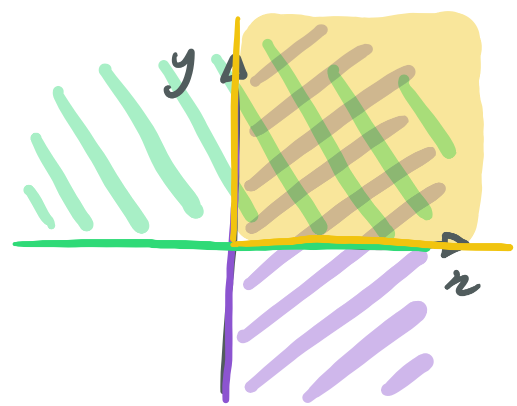
$\sqrt{x} \longrightarrow x \geq 0$

$\sqrt{y} \longrightarrow y \geq 0$



ANALOGAMENTE A PRIMA, AFFINCHÉ f SIA DEFINITA IN (x,y), LE DUE CONDIZIONI DEVONO ESSERE ENTRAMBE SODDISFATTE

$$D_f = \{(x,y) \in \mathbb{R}^2 \mid x \geq 0 \wedge y \geq 0\}$$



OK

3. $f(x,y) = \sqrt{x \cdot y}$

$$\sqrt{x \cdot y} \longrightarrow x \cdot y \geq 0 \quad \begin{cases} \nearrow & x \geq 0 \\ \searrow & y \geq 0 \end{cases}$$

DIVERSAMENTE DAI CASI PRECEDENTI, AFFINCHÉ f SIA DEFINITA IN (x,y), LE DUE CONDIZIONI DEVONO ESSERE ENTRAMBE SODDISFATTE O ENTRAMBE NON SODDISFATTE

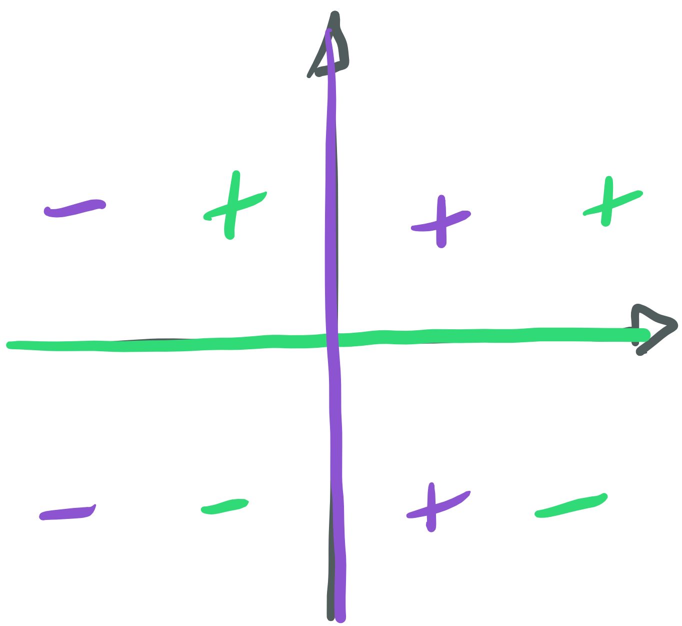
↳ DEVO FARE UNO STUDIO DEL SEGNO DI $x \cdot y$

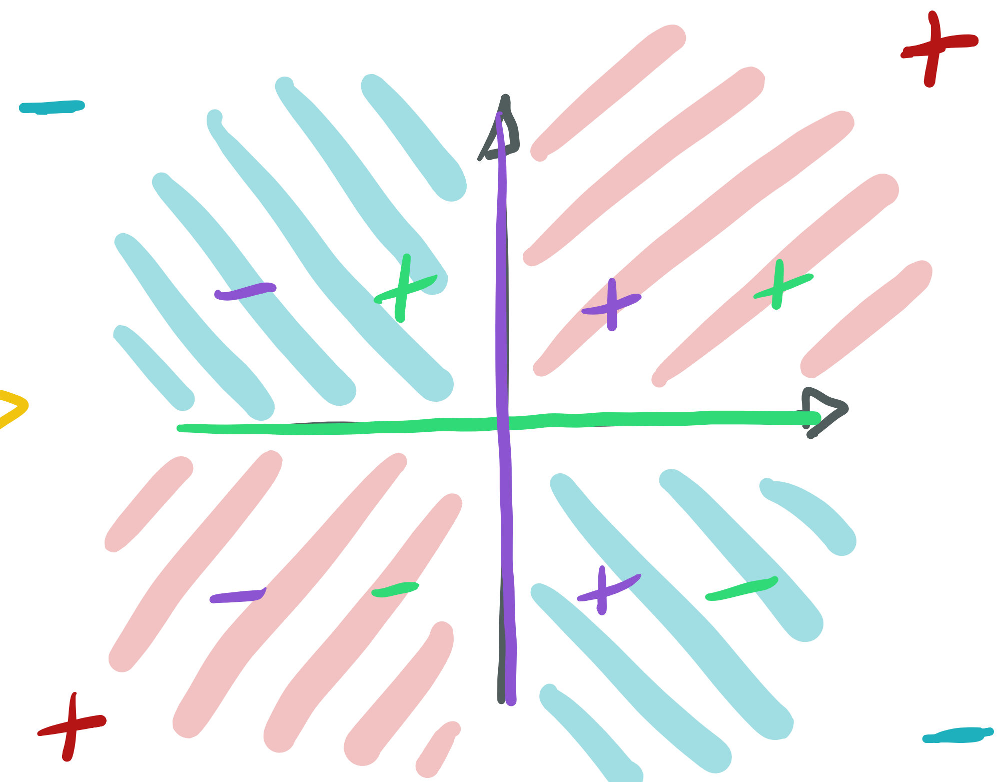
3. $f(x,y) = \sqrt{x \cdot y}$

$\sqrt{x \cdot y} \longrightarrow x \cdot y \geq 0$

$x \geq 0$

$y \geq 0$



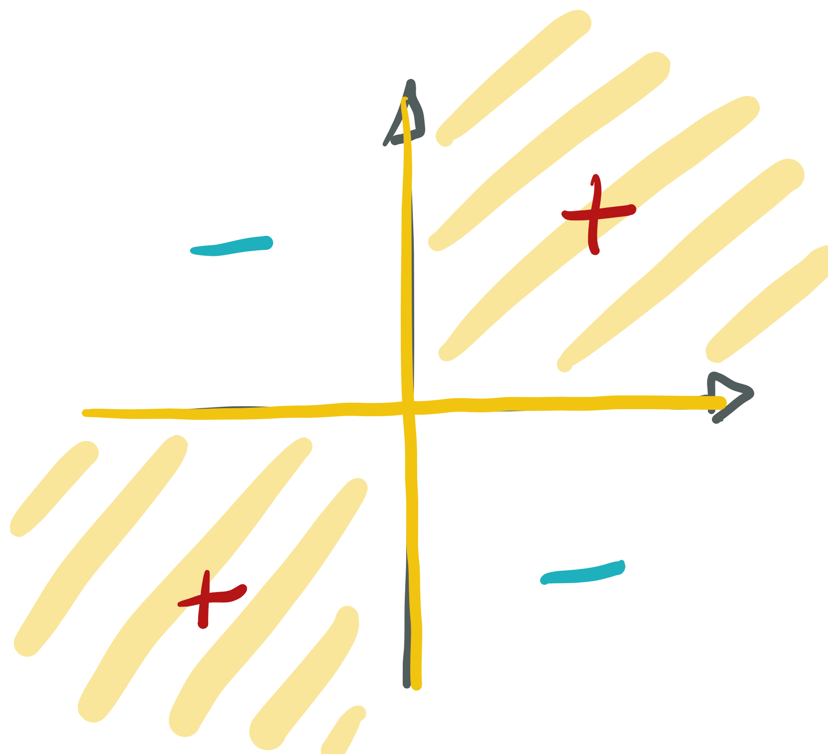


3. $f(x,y) = \sqrt{x \cdot y}$

QUINDI,

$$D_f = \{(x,y) \in \mathbb{R}^2 \mid (x \le 0 \wedge y \le 0) \vee (x \ge 0 \wedge y \ge 0)\}$$

CIOÈ,



OK

DOMINIO DI FUNZIONI DI PIÚ VARIABILI

TEOREMA DEI VALORI INTERMEDI

SIA $f : A \longrightarrow \mathbb{R}$ t.c.

- $A \subseteq \mathbb{R}^n$ é CONNESSO
- f é CONTINUA SU A

$\xRightarrow{\text{ALLORA}}$ $f(A)$ é UN INTERVALLO DI $\mathbb{R}$

CONSEGUENZA

SE $0 \notin f(A)$, PRESO UN QUALSIASI ELEMENTO $p_0 \in A$. ALLORA,

SE $f(p_0) > 0 \Rightarrow f(p) > 0 \quad \forall p \in A$

SE $f(p_0) < 0 \Rightarrow f(p) < 0 \quad \forall p \in A$

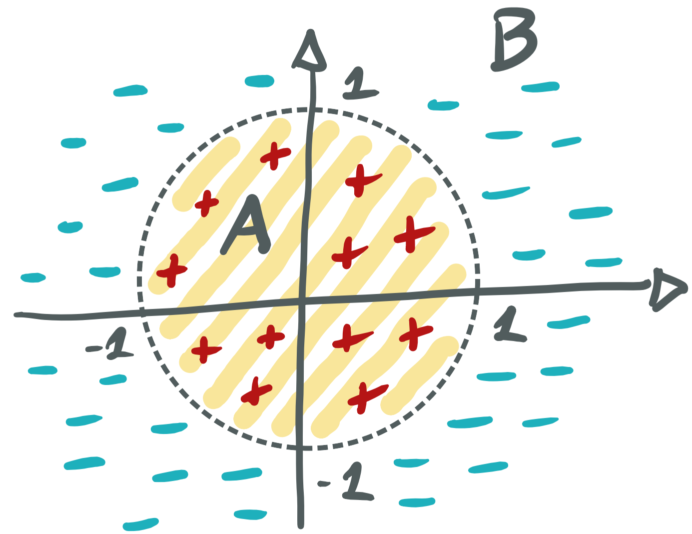
ESERCIZI

DETERMINARE L' INSIEME DI DEFINIZIONE DELLE SEGUENTI FUNZIONI:

1. $f(x,y) = \ln(1 - x^2 - y^2)$
2. $f(x,y) = \sqrt{2 - x^2 - y^2}$
3. $f(x,y) = \sqrt{-|x^2+y^2-2|}$
4. $f(x,y) = \ln\sqrt{-|x^2+y^2-2|}$
5. $f(x,y) = \sqrt{x^4 - y^2}$
6. $f(x,y) = \ln \frac{x^2-1}{1-y^2}$
7. $f(x,y) = \sqrt{x \sin\sqrt{x^2+y^2}}$
8. $f(x,y) = \sqrt{x}\sqrt{\sin\sqrt{x^2+y^2}}$

1. $f(x,y) = \ln(1 - x^2 - y^2)$

$1 - x^2 - y^2 > 0 \iff x^2 + y^2 < 1$



D_f = PUNTI INTERNI ALLA CIRCONFERENZA (BORDO ESCLUSO) DI CENTRO $(0,0)$ E RAGGIO 1

VOLENDO USARE IL TEOREMA DEI VALORI INTERMEDI, PONIAMO $g(x,y) = 1 - x^2 - y^2$

SIA A L'INSIEME DEI PUNTI INTERNI ALLA CIRCONFERENZA

$(0,0) \in A$, $g(0,0) = 1 > 0 \Rightarrow \forall (x,y) \in A,\ g(x,y) > 0$

$\Rightarrow D_f = A$

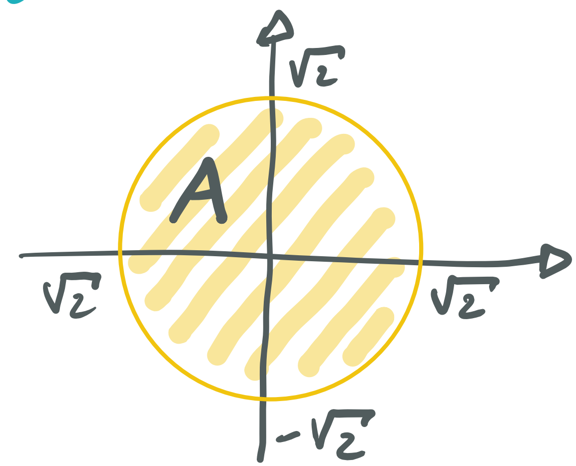
SIA B L'INSIEME DEI PUNTI ESTERNI ALLA CIRCONFERENZA

$(1,1) \in B$, $g(1,1) = -1 < 0 \Rightarrow \forall (x,y) \in B,\ g(x,y) < 0$

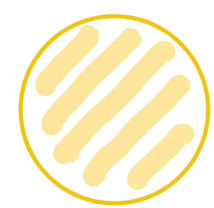
OK

2. $f(x,y) = \sqrt{2 - x^2 - y^2}$

$2 - x^2 - y^2 \geq 0 \iff x^2 + y^2 \leq 2$



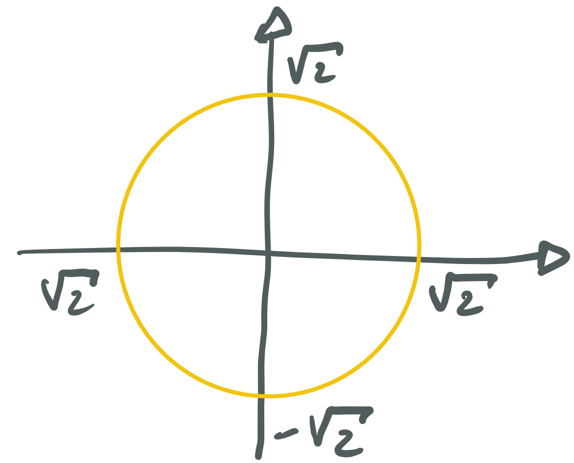
PUNTI INTERNI E BORDO DELLA CIRCONFERENZA DI CENTRO ORIGINE E RAGGIO $\sqrt{2}$

$D_f = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 2\} =$ 

OK

3. $f(x,y)=\sqrt{-|x^2+y^2-2|}$

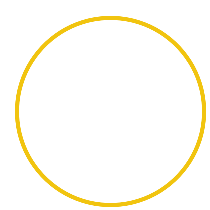
$$-|x^2+y^2-2|\geq 0 \iff x^2+y^2-2=0$$



$$\iff x^2+y^2=2$$

PUNTI DELLA CIRCONFERENZA
DI CENTRO $(0,0)$ E RAGGIO $\sqrt{2}$

$$D_f=\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2=2\}=$$



OK

4. $f(x,y) = \ln\sqrt{-|x^2+y^2-2|}$

$$\sqrt{-|x^2+y^2-2|} > 0 \iff -|x^2+y^2-2| > 0$$

NON ESISTE $(x,y) \in \mathbb{R}^2$ CHE SODDISFI TALE DISUGUAGLIANZA

$$D_f = \emptyset$$

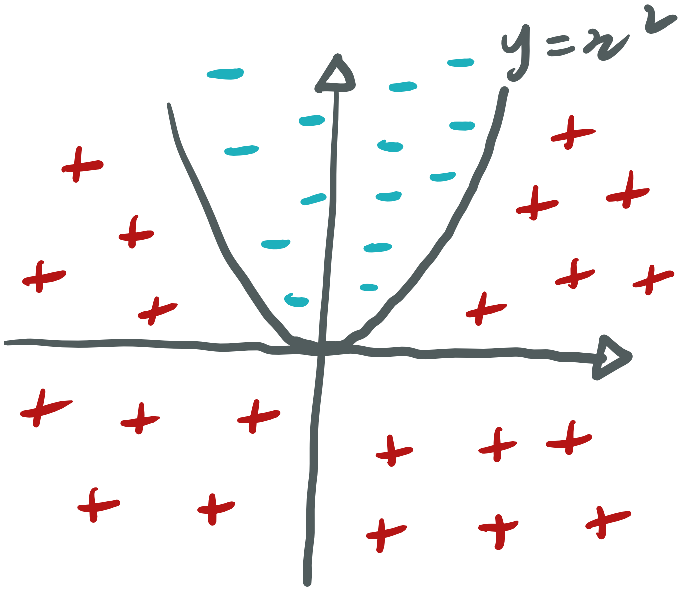
OK

5. $f(x,y) = \sqrt{x^4 - y^2}$

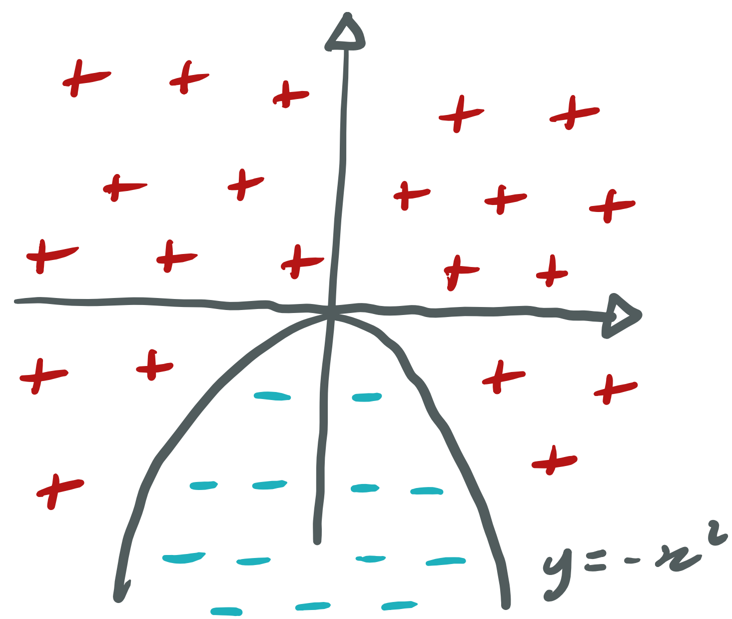
$x^4 - y^2 \geq 0 \iff (x^2 - y)(x^2 + y) \geq 0$

"STUDIO DEL SEGNO IN 2 DIMENSIONI"

$x^2 - y \geq 0 \iff y \leq x^2$

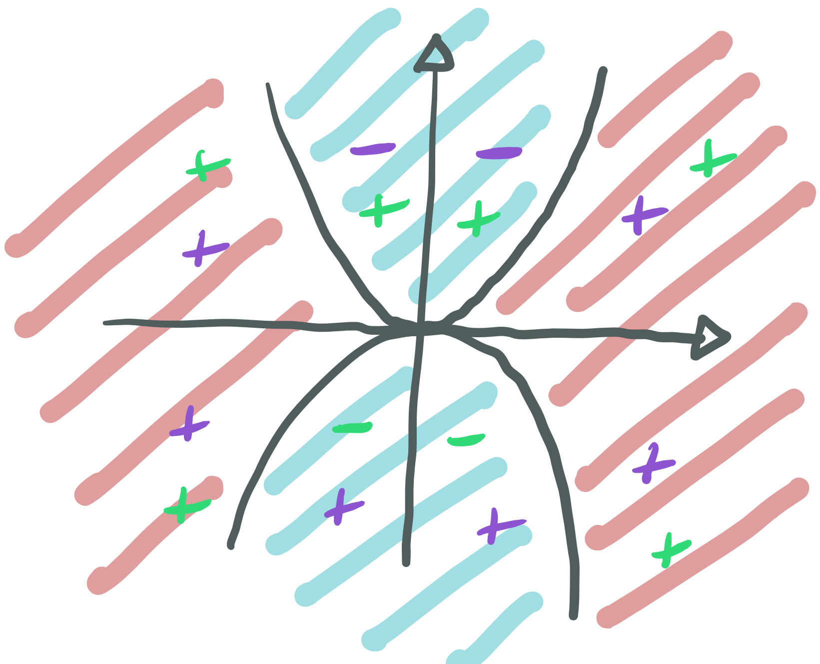


$x^2 + y \geq 0 \iff y \geq -x^2$



5. $f(x,y) = \sqrt{x^4 - y^2}$

"SOVRAPPONENDO" LE DUE IMMAGINI OTTENIAMO



$$(x^2 - y)(x^2 + y) \geq 0$$

$$\Downarrow$$

$D_f =$ 

OK

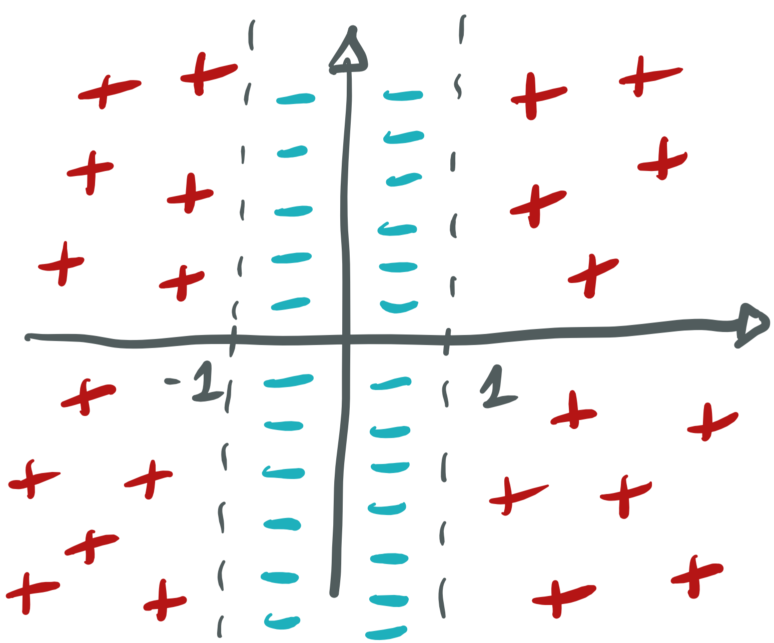
6. $f(x,y) = \ln \dfrac{x^2-1}{1-y^2}$

$$\frac{x^2-1}{1-y^2} > 0$$

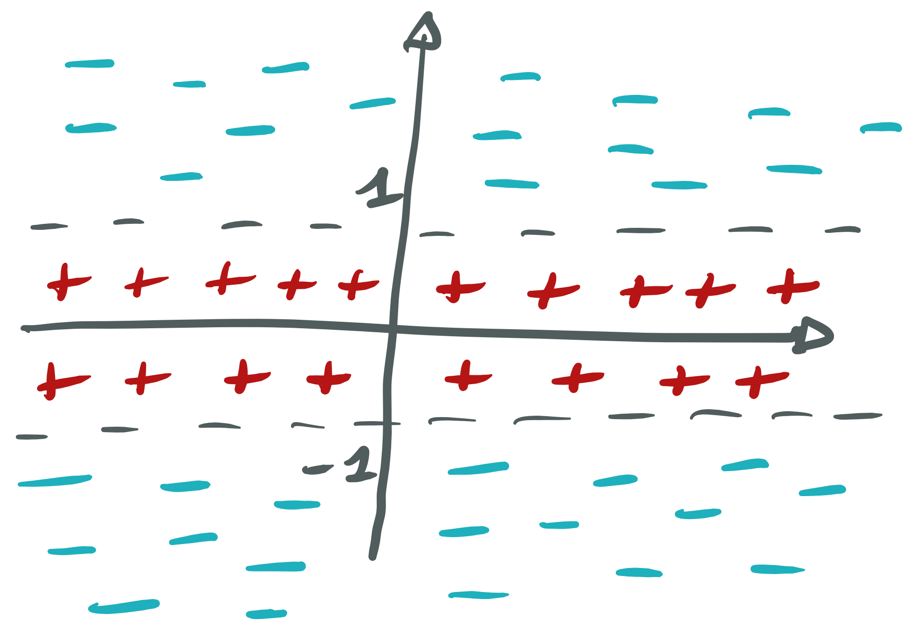
$x^2 - 1 > 0 \iff x^2 > 1 \iff x < -1 \vee x > 1$

$1 - y^2 > 0 \iff y^2 < 1 \iff -1 < y < 1$

NUMERATORE

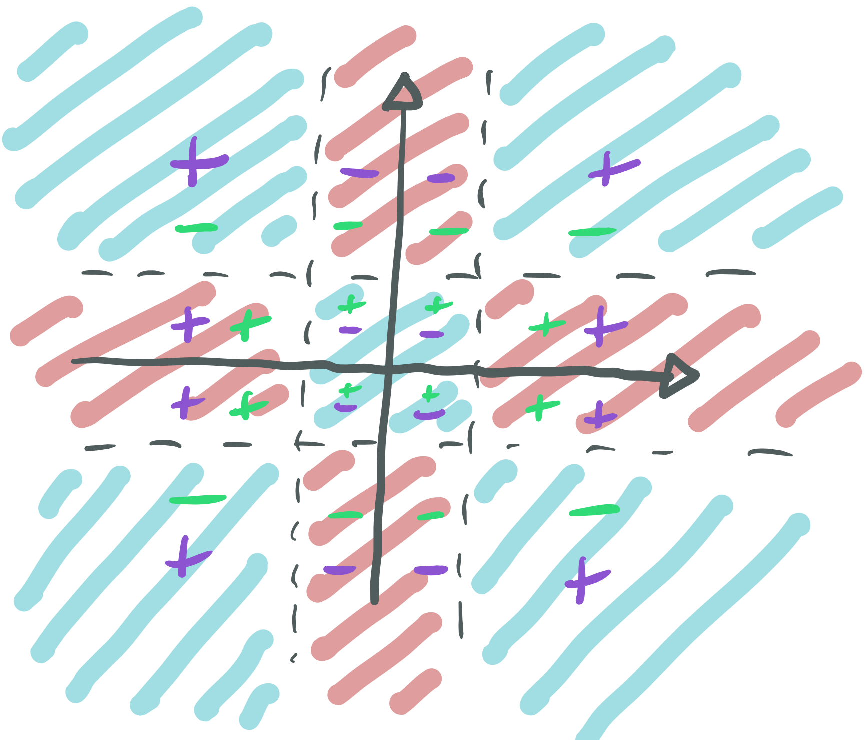


DENOMINATORE



6. $f(x,y) = \ln \dfrac{x^2-1}{1-y^2}$

"SOVRAPPONENDO" LE DUE IMMAGINI OTTENIAMO



$$\frac{x^2-1}{1-y^2} > 0$$

$$\Downarrow$$

$D_f =$

OK

7. $f(x,y) = \sqrt{x \sin\sqrt{x^2+y^2}}$

$x^2+y^2 \geq 0 \longleftarrow$ SODDISFATTA $\forall (x,y) \in \mathbb{R}^2$

&

$x \sin\sqrt{x^2+y^2} \geq 0 \longrightarrow$ $x \geq 0$

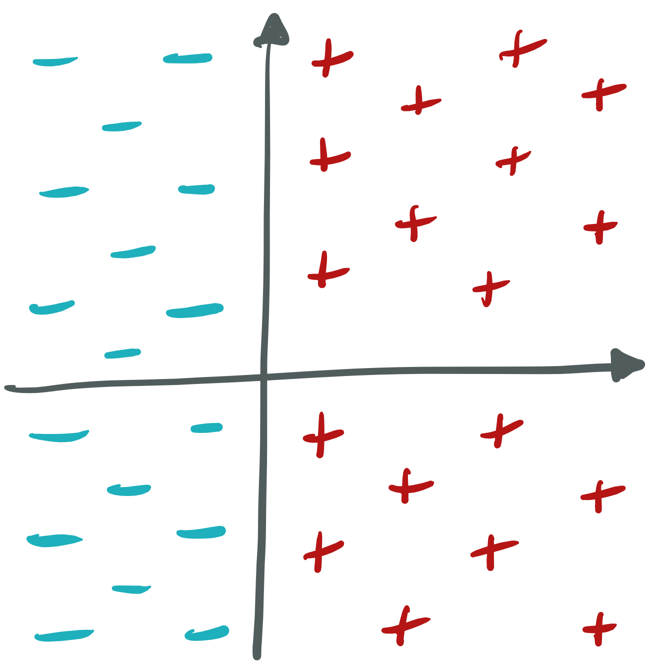
$\sin\sqrt{x^2+y^2} \geq 0$

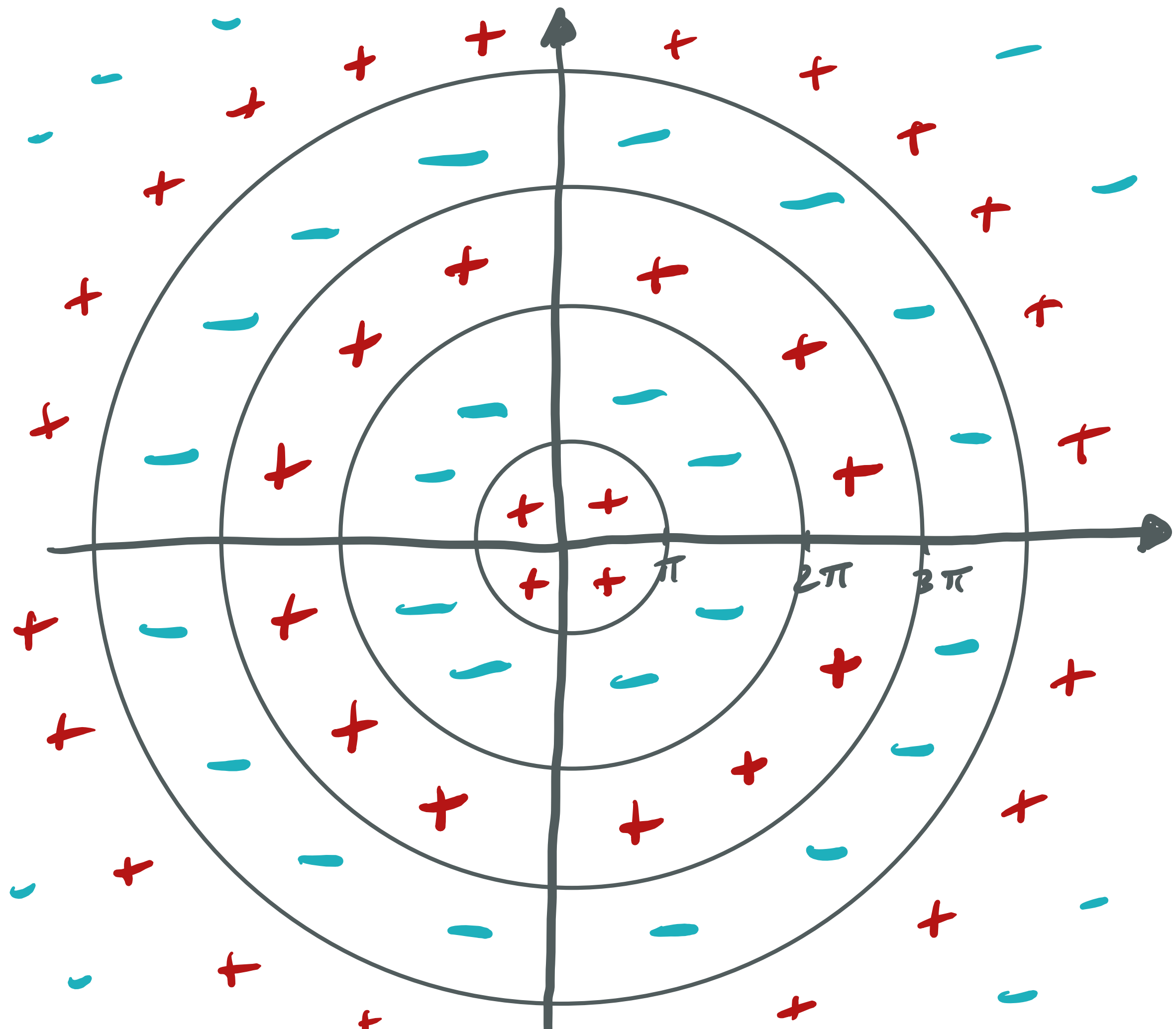
SODDISFATTA PER $0 + 2k\pi \leq \underbrace{\sqrt{x^2+y^2}}_{\substack{\parallel \\ \text{NORMA} \\ \text{DEL PUNTO} \\ (x,y)}} \leq \pi + 2k\pi$

7. $f(x,y) = \sqrt{x \sin \sqrt{x^2+y^2}}$

$$0 + 2K\pi \leq \sqrt{x^2+y^2} \leq \pi + 2K\pi$$

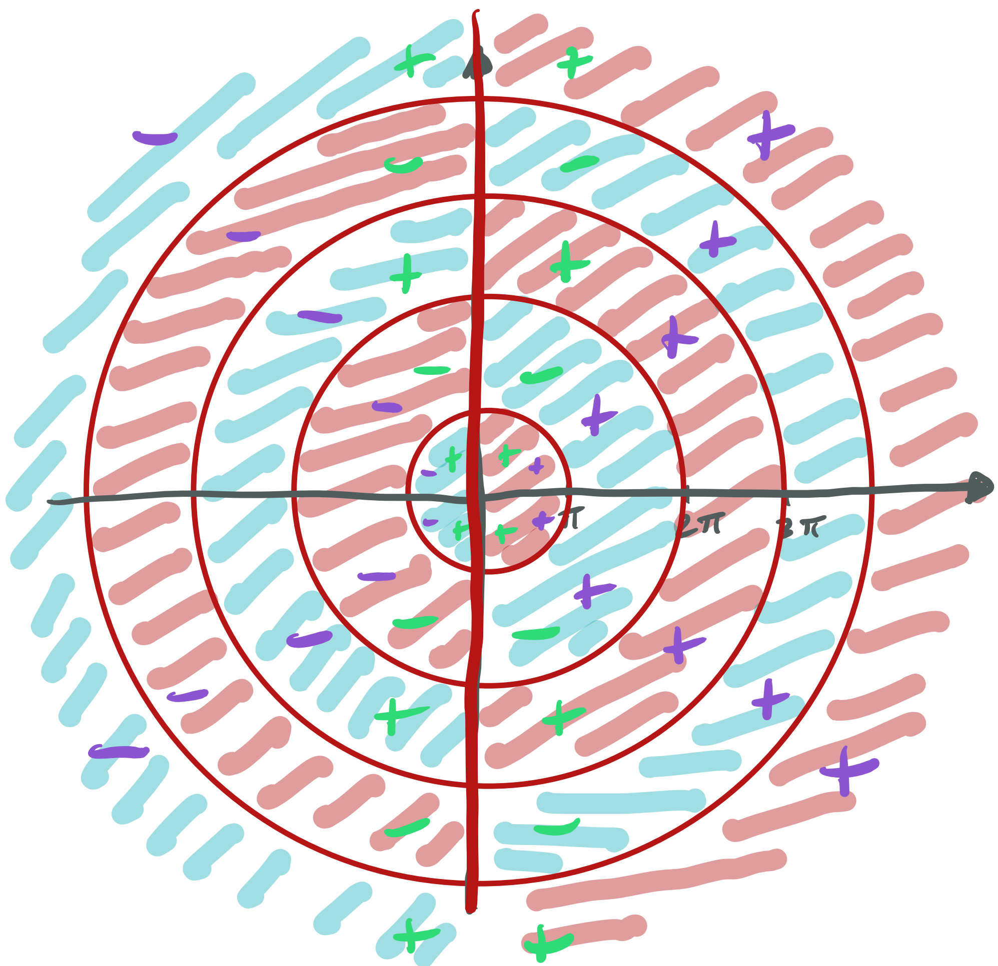
$x \geq 0$





7. $f(x,y) = \sqrt{x \sin\sqrt{x^2+y^2}}$

"SOVRAPPONENDO" LE DUE IMMAGINI OTTENIAMO



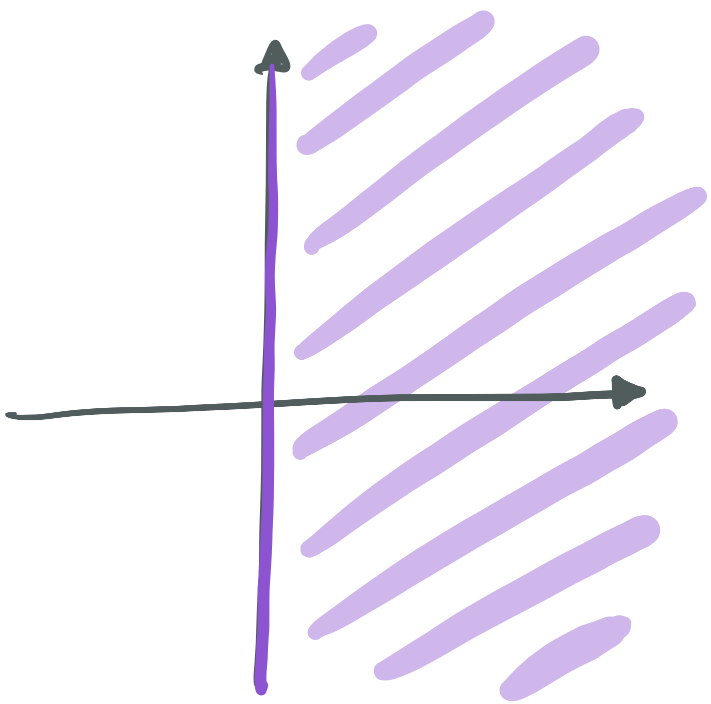
$D_f =$

OK

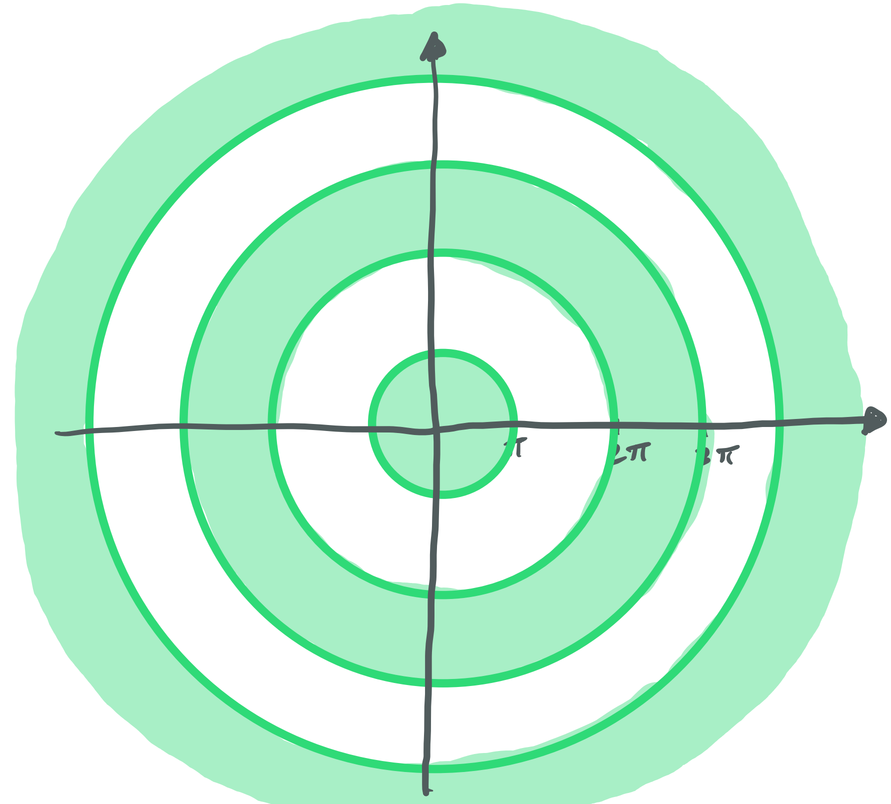
8. $f(x,y) = \sqrt{x}\sqrt{\sin\sqrt{x^2+y^2}}$

$\sqrt{x^2+y^2} \longrightarrow x^2+y^2 \geq 0 \longrightarrow$ sempre verificata

$\sqrt{x} \longrightarrow x \geq 0$

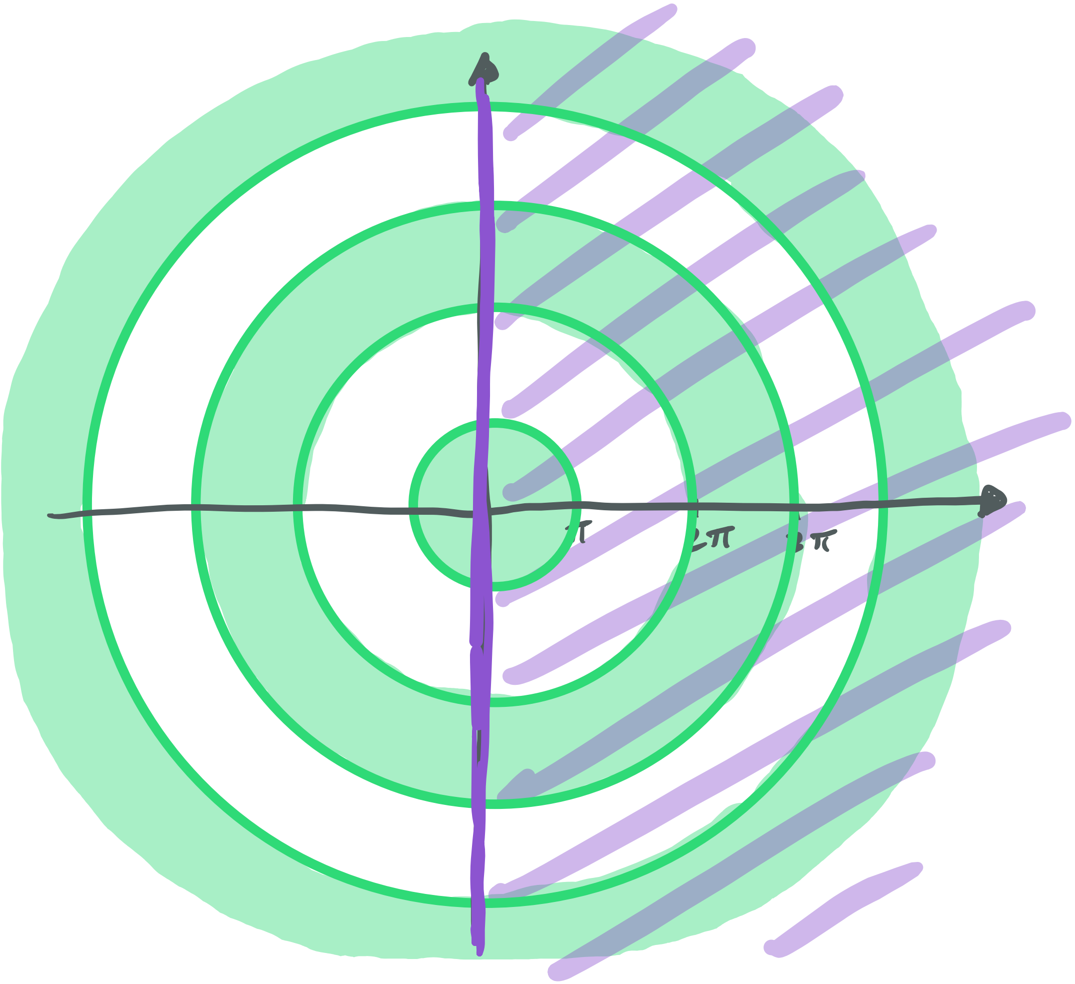


$\sqrt{\sin\sqrt{x^2+y^2}} \longrightarrow \sin\sqrt{x^2+y^2} \geq 0$

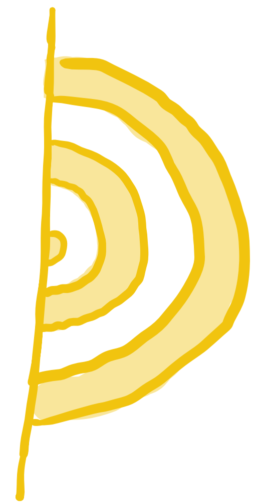


8. $f(x,y)=\sqrt{x}\sqrt{\sin\sqrt{x^2+y^2}}$

"INTERSECANDO" LE DUE IMMAGINI OTTENIAMO



$D_f =$



OK